XXX. *M.* DE LUC's *Rules, for the meaſurement of Heights by the Barometer, compared with Theory, and reduced to* Engliſh *Meaſures of Length, and adapted to* Fahrenheit's *Scale of the Thermometer: with Tables and Precepts, for expediting the practical Application of them. By* Samuel Horſley, *LL.D. addreſſed to Sir* John Pringle, *Bart. P. R. S.*

TO SIR JOHN PRINGLE, BART. P. R. S.

SIR,

Redde, Feb. 17, 1774.

IN the papers which I have the honor to preſent to you, nothing more was at firſt intended, than to reduce the *formulæ* given by Mr. DE LUC, in his elaborate work upon *the modifications of the atmoſphere*, for expreſſing differences of elevation, as indicated by the barometer and thermometer, in Paris toiſes, to others, which, from the like *data*, ſhould expreſs ſuch differences in Engliſh fathom; and at the ſame time be adapted to that ſcale of the thermometer, which is in general uſe in this country. Had I confined myſelf to this, which was my original

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ginal deſign, I ſhould certainly have ſuppreſſed whatever I might have executed, as I find my learned friend the ASTRONOMER ROYAL hath devoted ſome of his leiſure hours to theſe calculations; and the reductions, I propoſed, are actually performed, in a ſhort and elegant paper of his upon the ſubject, which is already, I believe, in Dr. MATY's hands. But I had made but a ſmall progreſs in my intended work, when it occurred to me, that *tables of the equations* for the effects of heat and cold, would be very uſeful. Such I have taken the pains to conſtruct, and have carried them to as great a length, as can be ever wanted. They are annexed to the enſuing piece, and will render the application of Mr. DE LUC's rules more eaſy and expeditious, than any peculiar diviſions of the thermometrical ſcale. When my tables were finiſhed, I thought it might be ſtill further uſeful, to give a ſuccinct explanation of Mr. DE LUC's orignal *formulæ*; that ſuch as have not leiſure to peruſe his excellent work, might be furniſhed with a competent idea of the reſult of his reſearches. This, I found, I could not do, in any way ſo ſatisfactory to myſelf, as by opening, as I went along, the principles of theory, in which the concluſions, he hath arrived at, appear to originate. And thus I was inſenſibly led into minute diſquiſitions, concerning the agreement of Mr. DE LUC's concluſions, from a long train of accurate experiments, with the geometrical theory of the atmoſphere, founded on the general laws of gravitation. In this manner, SIR, the papers before you have taken, as it were ſpontaneouſly, the form, in which they now appear; and a ſubject, remotely connected with my firſt deſign, takes up the far greater part of them. They can hardly be free from the imperfections naturally incident to productions, in which the plan hath been gradually changed, during the actual progreſs of the work. I flatter myſelf, however, that they are not deficient in two eſſential points, the preciſion and the perſpicuity of the mathematical reaſoning; and that, however unfiniſhed in ſome particulars, they are ſuch upon the whole, as the dignity of the ſubject may, in ſome degree, commend to your protection. There is perhaps no branch of phyſical enquiry, intrinſically more ſublime and intereſting, nor likely to be more important in its uſes, than that which immediately regards the conſtitution of that elaſtic fluid, which ſurrounds our globe, and appears to be a principal agent, in many of the moſt ſtriking operations of nature, and a neceſſary inſtrument, at leaſt, in carrying on the wonderful buſineſs of vegetation and of animal life. Upon a ſubject of ſo much importance, it muſt afford ſatisfaction, to find ſo exact an agreement, as is evinced, if I am not much deceived, in the enſuing pages, between a multiplicity of experiments, not ſuggeſted by any previous ſpeculations of theory, with a theory, whoſe concluſions in this branch had never before been duly ſubmitted to the teſt of experiment.

The whole of the following paper is divided into ſix ſections. The buſineſs of the firſt, is merely preparation for the principal work. It contains a brief account of the ſum of Mr. DE LUC's reſearches concerning the *variation of the heat of* *boiling water*, ſo far only as they reſpect the conſtruction and compariſon of mercurial thermometers; and it exhibits an actual compariſon of the ſcale of M. DE LUC's thermometer, with that which is likely to continue in general uſe here. In the ſecond, I ſtate the general principles of meaſuring heights by the barometer; or in other words, of the modification of the air, with reſpect to its condenſations, at different heights, by the preſſure of the ſuperior parts, excluſive of every other cauſe. Theſe principles are recapitulated for the ſake of perſpicuity and order; but, as they are generally well known, I refer to former writers for the demonſtration of them. This ſection concludes with the expoſition of M. DE LUC's fundamental rule. The third ſection treats of the equation depending on the difference of temperature of the quickſilver in the barometers, at the different places of ſynchronous obſervation. It explains, whence the neceſſity of this correction ariſes, and conſiders a caſe, in which the application of it requires ſome particular attentions; namely, the levelling a tract of flat country by the barometer. The fourth ſection explains the equation depending on the temperature of the air. This is likewiſe traced to its origin in theory; and upon this occaſion, I enquire into the condenſations of a fluid, unequally elaſtic in its different parts, under different degrees of compreſſive force: The fifth ſection contains the reduction of M. DE LUC's *formulæ*. I have found, with pleaſure, by the peruſal of Mr. MASKELYNE's paper, that we agree in our final concluſions. To this ſection, I have ſubjoined two problems; the one for eſtimating the variations which the denſity of the air, at any place, undergoes; and the other for determining the ſpecific gravity of the air of any temperature, at any elevation. The ſixth and laſt ſection points out certain conſequences, which ſeem to follow from the diſcoveries already made in conjunction with the theory eſtabliſhed. Theſe are only propounded to awaken curioſity, and promote enquiry. I am well aware, how little theory is to be truſted, in its *remote* concluſions, on account of the neceſſary deficiencies of the phyſical *data*, upon which its reaſonings are founded. The true uſes of it are, either to explain the mutual connexions and dependencies of things already known, or to ſuggeſt *conjectures* concerning what is unknown, to be tried by future experiment. And he who applies it, with due circumſpection, to theſe purpoſes, will always find it an uſeful engine. I flatter myſelf, that I have aſſigned the true cauſe of ſome very ſingular phænomena, remarked by M. DE LUC. I have endeavoured, to treat every part of my ſubject, in the plaineſt manner, that the nature of it would admit; and ſo diffuſely, as to be, I hope, intelligible to all, who are moderately well founded in the mathematical ſciences; for I have obſerved with pleaſure, that M. DE LUC's book hath raiſed a general curioſity upon the points it treats of; and for that reaſon, it ſeemed the more neceſſary, to depart in this inſtance, from a practice of late become too general, with mathematicians, *to write only for one another*. To which, if I am not miſtaken, it is in great meaſure owing, that theſe noble and uſeful ſtudies are leſs generally attended to, than they were in former times; when men of eminence, in this, as well as in all other branches, placed their principal glory, no leſs in communicating knowledge, than in acquiring, or ſeeming to poſſeſs it; and were content to dedicate large portions of their time, to the removing of thoſe difficulties for others, which they had once ſurmounted for themſelves. It is true, that in the variety of mathematical diſcuſſions, there are ſome of ſuch a nature, that it would be a deſperate attempt to render them intelligible to any but mathematicians; to thoſe, who to the natural faculty of combining, have added the mechanical and acquired habits of analytic calculation. Such ſubjects are beſt treated with ſcientific brevity. But this is no reaſon for wrapping up others in a ſimilar ſtile; and I muſt obſerve, that *concluſions*, at leaſt, are always ſuſceptible of ſimple perſpicuous expoſition, however abſtruſe the mode of inveſtigation, in ſome caſes, may neceſſarily be, by which they are brought out.

Such, SIR, is the general ſcope and plan of the following piece, which is preſented to you, in teſtimony of the author's reſpect and gratitude.

I have the honour to be, SIR,

your much obliged,

and moſt obedient ſervant.

Temple,
Nov. 18, 1773.

SAMUEL HORSLEY.

SECTION FIRST.

Of the VARIATION of the point of BOILING WATER, and the COMPARISON of THERMOMETERS.

THE degree of heat wherewith any fluid boils, is invariably the ſame, under a given preſſure; but if the preſſure be diminiſhed or increaſed, the boiling heat is diminiſhed or increaſed.

Water, placed under the exhauſted receiver, would be converted into ſteam, with a degree of heat, far inferior to that, which is neceſſary to its boiling in the open air; and under the preſſure of its own vapour, confined in PAPIN's digeſter, it is ſaid to ſuſtain a degree of heat, without boiling, far exceeding that, which, in the open air, would convert it into ſteam.

Hence it follows, that in climates, where the preſſure of the atmoſphere is liable to conſiderable change, the heat of boiling water, in the open air, will be different, at different times. Conſequently, thermometers, made in different ſtates of the barometer, will diſagree; unleſs allowance hath been made, for the effect of the variation of the barometer, upon accurate principles.

If care were taken to adjuſt the boiling point, to the mean height of the barometer, in every country, the inſtruments of the ſame country would always be conſiſtent; but thoſe of different countries would ſtill diſagree; that is, they would expreſs the ſame temperature differently, though their fundamental intervals ſhould be ſimilarly divided; for, in every ſcale, the number of degrees above or below melting ice, by which any given temperature is expreſſed, will be as the value of each degree inverſely; that is, if each be a given part of the fundamental interval, as the value of the fundamental interval inverſely; but, if the degrees of different ſcales be different parts of the fundamental intervals, as the value of the fundamental interval inverſely, and the number of degrees contained in it directly.

It is neceſſary here to explain ſome of my expreſſions. By the *fundamental interval* is to be underſtood the whole extent of the ſcale between melting ice and boiling water. This is not a particular *length*; for its length is not the ſame, even in thermometers made in the ſame ſtate of the barometer, unleſs their figures be equal and ſimilar; but it is a portion of the whole *ſolid content* of the thermometer, including ſo much of the tube, as reaches up to the point of melting ice; and it is the ſame portion of the whole ſolid content in all made in the ſame ſtate of the barometer; but in ſuch as have been made in different ſtates, a different portion in each. The degrees of the ſcale are ſubdiviſions of this portion of ſolid content, into leſſer aliquot parts. With reſpect to the number of theſe ſubdiviſions, the practice of different countries is different. By the *value* of the fundamental interval, or of the degrees of a thermometer, I mean the fractions, which expreſs what parts they are reſpectively of the whole ſolid content, terminated as above; that is, by the point of melting ice: and by the proportion of the fundamental intervals, or of the degrees, of different inſtruments, compared together, I underſtand the proportion of theſe fractions.

tions. Or every thing may be reduced to a compariſon of lengths, if in comparing the inſtruments of different countries, the compariſon be imagined to be made between two of equal dimenſions, and ſimilar figure, ſetting aſide that part of each which is above the point of melting ice.

To compare the thermometers, therefore, of different countries, the proportions of their fundamental intervals to each other muſt be aſcertained; or, we muſt have ſome means of finding, upon one ſcale, the place of the boiling point of another. For this purpoſe, a general ſolution is requiſite of the following problem: "*The fundamental interval* "*being given for a given height of the barometer,* "*to find the fundamental interval, for any other given* "*height of the barometer.*" The ſolution is furniſhed by M. DE LUC's laborious reſearches.

M. DE LUC fixes the boiling point of his thermometer, when the barometer is at 27 inches French [(a)], that being its mean height at GENEVA. He divides the fundamental interval, after the French manner, into 80 equal parts; and, by a great number of experiments, on the heat of boiling water, at different heights above the level of the ſea, the detail of which is to be found in his *Eſſai ſur la Variation*, &c: he hath found, that the height of his thermometer, plunged in boiling water, may be expreſſed, in all ſtates of the barometer, by the following *formula*, viz. $\frac{99}{200000} \log. y - a = T$. In which, y denotes the height of the barometer, in ſixteenths of a Paris line: T the height of a thermometer, plunged in boiling water, above melting ice, in hundredths of a degree of M. DE LUC ſcale; and a the conſtant number 10387 (b).

(a) Recherches, ſur les Modificat. de l'Atmoſphere, §. 451. a.

It is proper in this place to inform the reader, that M. DE LUC, by logarithms, always means the tabular or Briggian logarithms, and conſiders the 7 figures given by the tables, beſides the index, as integral figures; that is, he conſiders the eighth figure of the logarithm as ſtanding in the place of units. Throughout this paper, I have conformed myſelf to this manner of conceiving the tabular logarithms, that, upon the ſame ſubject, we may ſpeak the ſame language. But it is more uſual, with mathematicians, and, in general, it is more convenient, to conſider all the figures, after the index, as decimals. Thus the number, which M. DE LUC expreſſes by $\frac{99}{200000}$ log. y, would, in the common mathematical ſtile, be $\frac{99 \times 100}{2}$ log. y; or, 99×50 log. y.

It is but ſeldom that the barometer in this country ſtands ſo low as 27 French inches. 30 inches Engliſh are little more than its mean height upon the plane country in the neighbourhood of LONDON. It may, therefore, be proper for the London workmen to fix their boiling point, when the barometer is at 30 inches. FAHRENHEIT's diviſion of the ſcale, which makes 180 degrees between melting ice and boiling water, and places the point of 0 at the 32d degree below melting ice, may be retained; becauſe long uſe hath rendered it familiar to us. A thermometer thus conſtructed, I ſhall call BIRD's FAHRENHEIT: that eminent mechanic, our countryman, Mr. JOHN BIRD, having been, as I believe, the firſt workman, who took the pains to attend to the ſtate of the barometer, in making thermometers, and having always made it his practice, to fix the boiling point, when his barometer hath ſtood at 30 inches. Taking it for granted, that this ſcale will continue in general uſe here, I ſhall give an accurate compariſon of it with the ſcale of M. DE LUC.

(b) Ibid. §. 961. and §. 1143. note *a*. It may ſeem doubtful whether there is not a ſmall error in the conſtant number 10387. The experiments for aſcertaining the variation of the boiling point were made with a thermometer, of a peculiar ſcale; and the *formula* deduced from them was this, $\frac{7175}{30000000}$ log. $y - 5015$ = the height of the thermometer, plunged in boiling water, above melting ice, in parts of that peculiar ſcale. Recherch. ſur les Modificat. de l'Atmoſphere, §. 960. Now a degree of this ſcale was afterwards found to be to a degree of M. DE LUC's common ſcale as 80 to 3869¼. And this proportion between the degrees of the two ſcales, gives 10368,9 or 10369 very nearly, inſtead of 10387, for the value of the conſtant number a, in the *formula* for hundredths of a degree of the common ſcale.

T being put for the height of a thermometer plunged in boiling water, above melting ice, in hundredths of a degree of M. DE LUC's ſcale, in any given ſtate of the barometer; let Θ denote the ſame height, in hundredths of a degree of BIRD's Fahrenheit.

Put y for the height of the barometer in 16ths of a Paris line.
v for its height in Paris lines.
x in 10ths of a Paris inch.
z in 10ths of an Engliſh inch.

And for 10387 put a.
for 16 put b.
for 10 put c.
for 12 put d.

Alſo put E and F for numbers expreſſing the proportion of the Engliſh foot to the French foot.

Now M. DE LUC hath found, that whatever be the value of y;

$$\frac{99}{2000000}\,\text{log.}\,y - a = \text{T}.$$

$$\text{But log.}\,y = \text{log.}\,v + \text{log.}\,b.$$

$$\text{and log.}\,v = \text{log.}\,x + \text{log.}\,d - \text{log.}\,c.$$

$$\text{and log.}\,x = \text{log.}\,z + \text{log. E} - \text{log. F}.$$

$$\text{Therefore log.}\,y = \text{log.}\,z + \text{log. E} + \text{log.}\,d + \text{log.}\,b. - \text{log. F} - \text{log.}\,c.$$

$$\text{and } \frac{99}{2000000}\,\text{log.}\,z + \frac{99}{2000000}\,\overline{\text{log. E.} + \text{log.}\,d + \text{log.}\,b - \text{log. F} - \text{log.}\,c} - a = \text{T}.$$

$$\text{But } \frac{99}{2000000}\,\overline{\text{log. E} + \text{log.}\,d + \text{log.}\,b. - \text{log. F} - \text{log.}\,c} - a = -4171{,}55.$$

the French foot being to the Engliſh as 2,1315 to 2; *vide* Phil. Tranſ. vol. LVIII.

$$\text{Therefore } \frac{99}{2000000}\,\text{log.}\,z - 4171{,}55 = \text{T}.$$

And $\frac{99}{200000000}$ log. $z - 41{,}7155 = \frac{\text{T}}{100} =$ the height of the thermometer, plunged in boiling water, above melting ice, in degrees of M. DE LUC's ſcale, when the height of the barometer, in tenths of an Engliſh inch, is z.

Now for z write 300. Then $\frac{\text{T}}{100} = 80{,}902.$ which therefore is the height of the thermometer, in boiling water, above melting ice, in degrees of DE LUC's ſcale, when the barometer is at 30 inches Engliſh. And in the ſame ſtate of the barometer, the height of the thermometer plunged in boiling water, above melting ice, in degrees of BIRD's Fahrenheit, or, $\frac{\Theta}{100}$, is 180. Hence the numbers T and Θ are in the conſtant proportion of 809 and 1800, whatever be the value of z. For the change produced in the heat of boiling water, by any change of z, being always the ſame for both thermometers, the temperature expreſſed by T in parts of one ſcale, is always the ſame, as Θ expreſſes in parts of the other; and, therefore, putting $\frac{1}{L}$, and $\frac{1}{B}$, for the values of the hundredth part of a degree, of the ſcales of M. DE LUC and BIRD reſpectively, the fractions $\frac{T}{L}$, $\frac{\Theta}{B}$ are always equal, and T, Θ, are in the conſtant proportion of the invariable numbers L, B: conſequently, when the proportion of T and Θ is determined for any particular value of z, it is found generally for all. Therefore, as was affirmed,

$$T : \Theta = 809 : 1800.$$

And $T = \frac{809}{1800}\Theta = \frac{899}{2000}\Theta$ very nearly (c) in all values of z. and ſubſtituting this value, for T, in the equation exhibiting the relation between z and T, we ſhall have, for the relation between z and Θ,

$$\frac{99}{20000000}\log. z - 41{,}7155 = \frac{899}{2000 \times 100}\Theta.$$

Or, $\frac{99}{10000 \times 899}\log. z - 92{,}804 = \frac{\Theta}{100}$ = the height of the thermometer in boiling water, above melting ice, in degrees of BIRD's Fahrenheit, when the height of the barometer, in 10ths of an Engliſh inch, is z. And thus M. DE LUC's *formula*, for the variation of the boiling point, is adapted to Engliſh inſtruments, and reduced to Engliſh meaſures of length.

(c) It might be ſufficiently accurate for moſt purpoſes, to ſubſtitute $\frac{9}{20}\Theta$ ($= \frac{81}{180}\Theta$) for $\frac{809}{1800}\Theta$. The error of this ſubſtitution would be about $\frac{1}{809}\Theta$; and conſequently would amount to about $\frac{2}{9}$ of 1° of BIRD's Fahrenheit, when z is 300. But the error in the ſubſtitution I have uſed is much leſs, not amounting to $\frac{1}{8000}\Theta$, which makes leſs than $\frac{1}{44}$ of a degree of BIRD's ſcale, in the ſame caſe.

For z write 287,7525 (the length of 27 French inches in tenths of an Engliſh inch) and $\frac{\Theta}{100}$, the height of DE LUC's boiling point, above melting ice, in degrees of BIRD's Fahrenheit, comes out 177,989. Hence M. DE LUC's boiling point falls upon 209,989 of BIRD's ſcale; that is, upon 210 very nearly, or inſenſibly more than two degrees below BIRD's point of boiling; and the reduction of either ſcale to the other, in all inferior temperatures, will be as the table of compariſon ſhews.

By M. DE LUC's *formula*, thus reduced, the height of the thermometer, plunged in boiling water, above melting ice, in degrees of BIRD's Fahrenheit, in any given ſtate of the barometer, may be computed. But 899 being a troubleſome diviſor, to render the computation more eaſy and expeditious, take the following method. For $\frac{11}{1000000}$ log. z. write s.

Then $s + \frac{1}{900} s - 92{,}804 = \frac{\Theta}{100}$ very nearly. (d)

Upon theſe principles I have computed a little table, for finding the heights, to which a good BIRD's

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(*d*) If, according to note (*b*), we take 10369 inſtead of 10387, for the value of the conſtant number a in M. DE LUC's *formula*, we ſhall find, $\frac{99}{20000000}$ log. $z - 41{,}5355 = \frac{T}{100}$. Whence we ſhould obtain $s - \frac{1}{900} s - 92{,}198 = \frac{\Theta}{100}$ very nearly. But I abide by the *formula* given in the text: being perſwaded, that M. DE LUC hath purpoſely adopted the number 10387, as agreeing better, upon the whole, with his experiments than the other; though I do not recollect that he hath, in any part of his work, expreſsly ſaid ſo.

Fahrenheit will riſe, when plunged in boiling water, in all ſtates of the barometer, from 27 to 31 inches Engliſh. Among other uſes of this table, it will ſerve for a direction to inſtrument-makers, to make a true allowance, for the effect of the variation of the barometer, if they are at any time obliged, to finiſh a thermometer, when the barometer is above or below 30 inches; but in general it ſhould be their rule, to watch an opportunity of fixing the boiling point, when the barometer is actually at the height preſcribed.

I muſt, upon this occaſion, declare, how heartily I concur with M. DE LUC, in wiſhing, that ſome *common* ſcale, the ſame in the number of its diviſions, its point of 0, and its boiling point, might be received, with unanimous conſent, by philoſophers of all parts of the world; that, for the future, we might have one general language, for ſo very general an object of enquiry and diſcourſe, as the different degrees of heat and cold. To mathematicians the compariſon of different ſcales, is a taſk of little labour. But among thoſe who have a taſte for phyſical reſearches, and are capable of purſuing them, in certain branches at leaſt, with ſome degree of advantage to ſcience, as well as to themſelves, there are many, to whom every little calculation is a toil. Not having acquired the habits of it, in the early part of their lives, they never can acquire them, in ſuch a degree, as to be able to put a confidence in themſelves, and rely upon the accuracy of their own concluſions. The eaſe of ſuch perſons ſhould, in my opinion, be conſulted. But theſe are not the only perſons intereſted. The inconveniencies to be apprehended from a diverſity of ſcales are general. It is not one of the leaſt, that inſtruments eſſentially different, as made at different elevations in the atmoſphere, will continue to paſs under the ſame name. The error and confuſion, which this may create, is remarkably inſtanced, in what hath actually happened to the thermometer of the celebrated REAUMUR. The inſtrument, which at this day paſſes, all over Europe, under the name of REAUMUR's thermometer, is eſſentially different from his; yet it was always ſuppoſed to be the ſame, even among his own countrymen, till M. DE LUC detected the miſtake. At what time, upon what occaſion, or by whom, the change was introduced, is ſtill unknown.

SECTION SECOND.

Of the GENERAL PRINCIPLES of meaſuring HEIGHTS by the BAROMETER.

AS it is my deſign to compare the practical rules, which M. DE LUC hath deduced from experiment, with theory, it is neceſſary, for the clearer arrangement of the argument, previouſly to ſtate the general principles, upon which the meaſurement of heights by the barometer depends. In doing this, I ſhall rather aim at perſpicuity than at brevity; referring, however, for the demonſtration of whatever hath been advanced before, to writers of approved authority.

In any column of the atmoſphere, reſting perpendicularly upon any ſmall portion of the earth's ſurface, the denſities of the air diminiſh, as we aſcend to greater heights; and if the accelerative forçe of gravity were the ſame at all heights, the denſities would decreaſe geometrically as the height increaſed arithmetically. This is an obvious conſequence from a known property of air, that it expands itſelf through a greater or a-leſs ſpace, in proportion as the force, by which it is compreſſed, is leſs or greater; that is, that the denſity of air, is always as the compreſſing force (*e*). And from hence it would follow, that the difference of the elevation of any two places, would be as the logarithm of the *ratio* of the denſities of the air, at each: and the denſity being every where as the compreſſing force, and the compreſſing force as the length of the column of quickſilver ſuſtained by it in the barometer, the difference of elevation would be, as the logarithm of the *ratio* of the altitudes of the quickſilver in the barometer, at the ſame time, at the different ſtations; that is, as the difference of the tabular logarithms of the numbers, by which thoſe altitudes would be expreſſed in any given meaſure (*f*). But the accelerative force of gravity diminiſhes, in the ſame proportion, as the ſquare of the diſtance, from the earth's center, is increaſed. It is not the ſame

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(*e*) Cotes's Hydroſtat. Lectures, lect. ix.

(*f*) See Philoſoph. Tranſact. n. 181. Phil. Nat. Princip. Math. lib. ii. prop. 22. Scholium. Cotes's Hydroſtat. Lectures, lect. ix. Harmon. Menſ. p. 17.

Of all theſe demonſtrations, that given by Mr. COTES, in the hydroſtatical lectures, will be the moſt perſpicuous to the generality of readers. It is very diffuſe, and he hath been at great pains to reduce it to the moſt ſimple principles. Mathematicians will find the ſubſtance of his argument well ſummed up, by M. DE LA LANDE, in the Connoiſſance for the year 1765, p. 211, 212. who, I am perſwaded, would not have produced it as a new demonſtration, had he known it had been given before.

therefore at all heights above the ſurface; and the denſities of the air muſt decreaſe, by a different law, from that which would obtain, if the force of gravity were uniform. This other law, however, is ſuch, that, to a much greater height than is acceſſible to man, the gradual variation of the compreſſing force and denſities of the air, will be ſo little different, from what it would be upon the former hypotheſis, that the error of that hypotheſis, in the meaſurement of heights, will be abſolutely inſenſible.

Let the point C [tab. IX. fig. 1.] repreſent the center of the earth. CA the earth's ſemi-diameter. AB any height above the ſurface. At A, place a right line, AD, of any finite length, at right angles with AC. In the right line AC, towards C, take Aβ, ſuch that CA may bear to Aβ the proportion of CB to BA. In a right line drawn through β, at right angles with AC, take βE, of ſuch length, as to bear to AD the proportion of the denſity of the air at B to the denſity at A, or at the earth's ſurface. The curve, which the point E always touches, is a logarithmic, of which AC is the aſymptote (g).

As I ſhall have frequent occaſion to conſider the curve, which thus exhibits the relation between denſity and elevation, I ſhall call it the ATMOSPHERICAL LOGARITHMIC.

Imagine this curve deſcribed, and take another height Ab, and take Aϐ $= \frac{CA \times Ab}{Cb}$, and draw ϐ$e$

(g) Cotes's Hydroſtat. Lectures, p. 161—167. Harmon. Menſ. p. 18. Phil. Nat. Princip. Math. lib. ii. prop. 22. Brooke Taylor, Method. Increment, Prop. 26.

parallel to βE, meeting the curve in *e*. Then $\beta\epsilon$ is the logarithm of the *ratio* of βE to ϵe, or of the denſity at B to the denſity at *b*. But if the greater of the two heights, AB and A*b*, bear but a very ſmall proportion to the ſemi-diameter of the earth, their difference B*b* will be very nearly equal to $\beta\epsilon$.

For, becauſe CB : BA $=$ CA : Aβ (by conſtruction.)
Therefore, by converſion, CB : CA $=$ CA : Cβ.
In like manner, and by inverſion, CA : C*b* $=$Cϵ : CA.
by equi-diſtance perturbate, CB : C*b*$=$Cϵ : Cβ.
and converting, CB : B*b*$=$Cϵ : $\beta\epsilon$.
by permutation, B*b* : $\beta\epsilon$$=$CB : C$\epsilon$.

But when AB is infinitely diminiſhed, CB$=$CA ultimately. Alſo A*b* being infinitely diminiſhed, Cϵ $=$ CA ultimately. Therefore CB $=$Cϵ ultimately, and B*b* $=$$\beta\epsilon$ ultimately. Q. E. D.

Now AB and A*b* will always be ſo ſmall, with reſpect to CA, if B and *b* be ſuppoſed to repreſen any acceſſible places, that CB, Cϵ, and B*b*, $\beta\epsilon$, may always, in this caſe, be conſidered, as in their ultimate proportion of equality.

It is ſtill therefore to be admitted as a principle, in practice, that the difference of elevation of any two places, is as the difference of the tabular logarithms of the heights of the quickſilver in the barometer at the ſame time, at both places; that is, it is the logarithm of the *ratio* of thoſe heights in *ſome* ſyſtem of logarithms. And the heights of the quickſilver being given, by obſervation, the difference of elevation will be known, if that particular ſyſtem can be determined; that is, if the *modulus* of the ſyſtem, or the length of the ſubtangent of the curve DE*e* of that ſyſtem, can be aſcertained, in ſome known meaſure, as Engliſh fathoms, or Paris toiſes.

The eaſieſt method of doing this, that theory ſuggeſts, is to compare barometers at two ſtations, ſuppoſe B and *b*, each of a known elevation AB and A*b*, above the level of the ſea. For the logarithms of any given *ratio*, in different ſyſtems, are proportional to the ſubtangents; and the difference of elevation, B*b*, diminiſhed in the proportion of CB, (the diſtance of the higher ſtation from the earth's center) to Cϐ, (a third proportional to C*b*, the diſtance of the lower ſtation from the earth's center, and CA, the earth's ſemi-diameter) is the logarithm of the *ratio* of the denſity at B, to the denſity at *b* (that is, of the columns of quickſilver ſuſtained in the barometer at B and *b*) in the atmoſpherical ſyſtem. Therefore, as the difference of the tabular logarithms, of theſe columns, to the ſubtangent of the tabular ſyſtem, ſo ſhould B*b*, diminiſhed as hath been ſaid, (that is, ſo ſhould βϐ) be to the ſubtangent of the atmoſpherical logarithmic. The utmoſt height, to which we can aſcend, above the level of the ſea, is ſo ſmall, that the reduction of B*b* may, even in this inveſtigation, always be neglected. For, if AB were four Engliſh miles, which exceeds the greateſt acceſſible heights, even of the Peruvian mountains, and Aϐ three, βϐ would be ſcarce one part in 500 leſs than B*b*. So that, by comparing barometers at different elevations, within a mile above the level of the ſea, the ſubtangent of the atmoſpherical curve might be determined, as it ſhould ſeem, without ſenſible error, by taking ſimply the difference of elevation, without reduction, for the logarithm of the *ratio* of the obſerved heights of the quickſilver in the atmoſpherical ſyſtem. Thoſe, however, who have attempted to determine the ſyſtem, by this method, have hitherto agreed but ill in their concluſions. The fact is, that the length of this ſubtangent is very different at different times. The cauſes and quantity of its variation will be conſidered in another place. It appears, from M. DE LUC's experiments, that, though otherwiſe ſubject to change, it is conſtant in a given temperature. And that when the temperature of the air is + 16$\frac{3}{4}$ of his ſcale, the difference of the tabular logarithms of the heights of the quickſilver in the barometer, gives the difference of elevation in 1000ths of a Paris toiſe [b]. This is the rule, which he hath derived from a great number of experiments made at very different elevations: and the truth of it being admitted, it is a neceſſary conſequence, that the number, which is the *modulus* of Briggs's ſyſtem, expreſſes the length of the ſubtangent of the atmoſpherical curve, ſuch as it is in that temperature, in 1000ths of a Paris toiſe.

(*b*) Recherch. ſur les Modif. de l'Atmoſph. §. 588.

SECTION THIRD.

Of the EQUATION for the DIFFERENCE of TEMPERATURE of the QUICKSILVER in the BAROMETERS, at the DIFFERENT STATIONS.

THE preceeding rule, however, muſt not be expected to give an accurate reſult, even in that particular temperature of the atmoſphere, to which it is adapted, unleſs the ſpecific gravity of the quickſilver, in the barometer, hath been the ſame at both ſtations, at the time of obſervation. If the ſpecific gravity hath been different, at the different ſtations, the heights of the quickſilver, in the barometer, will *not* have been proportional to the denſities of the air; that is, to the forces by which they have been ſuſtained: and the calculation, built upon the ſuppoſition that they were ſo, becomes erroneous. The ſpecific gravity of every material ſubſtance varies with its temperature. If the temperature of the quickſilver, therefore, hath been different, at the different ſtations, the difference of elevation, found by the foregoing rule, will require correction, though the mean temperature of the air may have been ſuch as it preſcribes (i). No particular temperature of the quickſilver is neceſſary to the accuracy of the reſult of the preceeding rule, or to render the correction, I am now conſidering, 0; but only, that its temperature, whatever it be, be the ſame at both ſtations. If the *temperature* hath been the ſame, the *ſpecific gravity* hath been the ſame; and if the *ſpecific gravity* hath been the ſame, the *length* of the columns of quickſilver have been as the forces by which they were ſuſtained, whatever the common ſpecific gravity may have been. Columns of water, ſuſtained in evacuated tubes, of ſufficient length, would be proportional to columns of quickſilver in the barometer, at the ſame times and places, provided the temperature of the two water columns were the ſame, and that of the two mercurial columns the ſame; and, conſequently, the difference of the logarithms of the water columns would be preciſely the ſame as of the mercurial columns. For the water columns, and the mercurial columns, are only expreſſions of the ſame abſolute magnitudes, the forces by which they are both ſuſtained, in parts of different ſcales. But if the temperature of the quickſilver, or of the water, be different at the ſame time, at different ſtations; then, though we compare the water columns with each other, and the mercurial columns with each other, ſtill we compare different things, though we call them by the ſame name. We compare fluids of different ſpecific gravities; that is, we meaſure the preſſure of the air at one place, in parts of one ſcale, and at the other place, in parts of another. The error is of the ſame kind, as if one ſhould attempt to determine the proportion between two parts of a building, by meaſuring one with a Paris foot-rule, and the other with an Engliſh foot-rule, without attending to the difference between Paris feet and London feet; and the ſame method muſt be taken to get rid of the error in both caſes: we muſt aſcertain the difference of the ſcales we have applied, and make allowance for it: we muſt know the length of the one in parts of the other; that is, in the particular caſe in queſtion, we muſt be able to determine, from the obſerved height of the barometer, when the quickſilver is of any given temperature, what its height would have been, at the ſame place, and at the ſame time, if its temperature had been any other, that may be aſſigned. I have thought it neceſſary to be thus minute, in explaining the principles, upon which the correction in queſtion depends; becauſe it is a point, which is likely to be miſunderſtood, though in itſelf of no great difficulty. M. DE LUC himſelf hath fallen into a miſtake, not with reſpect to the quantity of the error, but the manner of allowing for it; which, however, is of no other bad conſequence, than that of lengthening the calculation unneceſſarily. He imagines that a particular temperature of the quickſilver is neceſſary, that the error ſhould be nothing; and to this he always reduces the obſerved heights of the quickſilver, in the barometer, at both ſtations (k). But the reſult of the computation would, in all caſes, have been the ſame, if they had been reduced to any other given temperature; and therefore, it is always ſufficient, to reduce the one to the temperature of the other. This little overſight I ſhould have touched upon with more reluctance, were it not of too little importance to derogate, in the leaſt degree, from the general merit of M. DE LUC's elaborate and invaluable work.

(i) Obſerve, that the temperature of the quickſilver, in the portable barometer, will not be neceſſarily the ſame with that of the circumambient air, at the place and time of obſervation; there will generally be a conſiderable difference.

(k) Recherch. ſur les Modif. de l'Atm. §. 369—374.

The quantity of the correction in queſtion, is thus determined. At times, when the barometer, in a temperate air, ſtood at 27 Paris inches, M. DE LUC tried what change was effected in its height, by changes of temperature, induced by art in the quickſilver, without any alteration in the ſtate of the atmoſphere; and, by repeated experiments of this kind, it was found, that the difference between the length of the column of quickſilver, ſucceſſively heated to the temperature of boiling water, and cooled to that of melting ice, amounted to half a Paris inch exactly; that is, to $\frac{1}{54}$th of the mean height (l). But the whole extent of the thermometer's ſcale, from the temperature of melting ice to M. DE LUC's boiling point, being 178° of BIRD's Fahrenheit, the change of the height of the barometer, due to 1° of BIRD's Fahrenheit is always $\frac{1}{54 \times 178} = \frac{1}{9612}$. And if n denote the number of degrees of BIRD's Fahrenheit, in the difference of the temperatures of the quickſilver, in the barometers, at different ſtations, the reduction will be $\frac{n}{9612}$; that is to ſay, the height of the warmer column muſt be ſhortened by this part of its own length, or that of the cooler augmented by the like part, to reduce either, to the height it would have ſtood at, at the time and place of obſervation, with the temperature of the other.

(l) Recherch. ſur les Modif. de l'Atm. §. 362—364.

Theſe determinations of the effect of heat upon the column of quickſilver, in the Torricellian tube, ſhew the proportional alterations of the denſity of that fluid, by given increments, or diminutions, of heat. For the perpendicular height of a column, of any inelaſtic fluid, ſuſtained in the Torricellian tube, by a given compreſſive force, muſt, by the known laws of hydroſtatics, be as the denſities of the fluid inverſely. And as this proportion muſt obtain, whatever be the ſize, or figure, of the tube, there ſeems to be no method, by which the change of denſity, or the proportional expanſion of quickſilver by heat, can be meaſured with more preciſion. Theſe concluſions, therefore, may be of uſe in many phyſical enquiries; and there are many caſes, in which it may be neceſſary, to reduce the obſerved height of the barometer, in one temperature, to another. Thus when that height is to be made the meaſure of the variable preſſure, or of the denſity of the air, in ſome particular place, it will be neceſſary to chooſe ſome *ſtandard* temperature, to which the obſerved length of the column may always be reduced. And it was this conſideration, as I have gathered from many converſations with him, which gave occaſion to M. DE LUC's miſtake. He had ſettled it with himſelf, at his very firſt entrance upon theſe reſearches, that the point he was to keep conſtantly in view, as the ultimate object of his whole purſuit, was to find, in the variable length of the mercurial column, a meaſure of the preſſure and denſity of the air. This, he ſaw, was only to be looked for in quickſilver of ſome one conſtant temperature; and thus he became poſſeſſed with a general notion, of the neceſſity of reducing the obſervations of the barometer, to ſome ſtandard temperature, upon all occaſions.

But, for the particular purpoſe of computing differences of elevation, inſtead of attending to the correction of the obſerved heights of the quickſilver at all, it will be a readier way, to make the correſponding correction immediately, upon the difference of the tabular logarithms. If any quantity be diminiſhed or increaſed, in a given proportion, the logarithm of its proportion to any other given quantity (that is, the difference of the tabular logarithms) is diminiſhed or increaſed by a given magnitude; namely, by the logarithm of the given *ratio*, in which the variable quantity is altered. The logarithm of the *ratio* of x to $x + \frac{1}{9612} x$, or of 9612, to 9613, is 452 (m). Therefore, if n, as before, expreſs the difference of temperature in degrees of BIRD's Fahrenheit, $\overline{n \times 452}$ is the correction to be applied to the difference of the logarithms of the obſerved heights of the quickſilver in the barometer. This correction is to be ſubſtracted, if the temperature, at the upper ſtation, hath been the cooler of the two, and to be added, in the contrary caſe. I am indebted to the ASTRONOMER ROYAL for the firſt hint of this elegant method of applying the correction in queſtion.

(m) The reader will here recollect, that I ſpeak of the tabular logarithms in the language of M. DE LUC. The logarithm of the *ratio* of 9612 to 9613 is, in the more common ſtile, 0.0000452.

The third of the annexed tables, ſhews the quantity of this correction correſponding to every value of *n* from 1° to 70°. The third column, to the right, exhibits the value of it in Engliſh fathom, in that particular temperature of the air, in which the difference of the tabular logarithms of the heights of the quickſilver in the barometer, gives the difference of elevation in 1000ths of an Engliſh fathom. What that temperature is, will be determined in the ſequel. It is to be obſerved, that the value of theſe corrections, in the fathom of any other country, will be the ſame, in that particular temperature, in which the tabular ſyſtem meaſures the difference of elevations in 1000ths of the fathom of that country.

The rule, which I have given for applying this correction, ſuppoſes that it is previouſly known, which of the two ſtations is the higheſt; otherwiſe it will be doubtful, whether it ſhould be added or ſubtracted. This doubtful caſe may actually happen in levelling a conſiderable tract of country, that is pretty even. The table, however, will ſtill give the quantity of the correction. Add that quantity to the logarithm of the height of the cooler barometer; and the difference between this logarithm, ſo augmented, and the logarithm of the height of the other, is the difference of the logarithms of the obſerved heights, duly corrected: and the ſtation of the cooler barometer, was the lower of the two, if its augmented logarithm exceeds that of the other: in the contrary caſe, the higher.

It hath been already obſerved, that the change of the denſity of quickſilver, by an alteration of its heat, muſt always be proportional to the increaſe or contraction of the length of the column in the barometer; or, in other words, that the alteration, in the perpendicular height of the Torricellian column, is proportional to the change of the *ſolid content* of ſpace, occupied by a given quantity of the fluid. Thoſe who recollect the experiments of the celebrated BOERHAAVE, for meaſuring this change of volume, cannot but be ſtruck with the great and ſingular diſagreement between his concluſions and thoſe of M. DE LUC. BOERHAAVE makes the whole expanſion, produced by a change of temperature from the 0 of Fahrenheit, (or the forced congelation of *ſal ammoniac*) to the heat of boiling water, $\frac{1}{52\frac{25}{53}}$ of the whole volume, which is very little more than M. DE LUC found to be produced by a change from 32° of Fahrenheit (or the diſſolution of natural froſt) to the heat of boiling water (n).

(n) Vid. Elementa Chemiæ, vol. 1. p. 174. In this place, the author expreſsly treats of the expanſion of quickſilver by heat; and the proportion, which he here aſſigns, hath been adopted by the writers of elementary phyſics. But, p. 165, having deſcribed an artificial cold, in which the thermometer had been ſunk to −40 of Fahrenheit, he ſays, "*Novimus hoc* "*argenti vivi corpus ab illo gradu* 40 *infra* 0, *ad gradum* 600 *quo* "*incipit ebullire contractum fuiſſe per partes* 640 *totius molis* "10782." And, in the explanation of one of his copper plates, he deſcribes a thermometer in which the quickſilver, in the greateſt natural cold, juſt filled the bulb, which contained, he ſays, 11520 ſuch parts as the tube contained 96. It ſeems probable, that the number 11124, which is given, p. 174, as the whole volume of the quickſilver in the temperature 0, (a degree of Fahrenheit being the unit) hath been deduced from the mean of a variety of experiments upon different inſtruments, for it differs not greatly from the mean of the two numbers 10782 and 11520.

Upon this it is to be obſerved, that BOERHAAVE's method gave him the expanſion of his quickſilver, diminiſhed by the whole expanſion nearly of the veſſel which contained it. DE LUC's gave him the expanſion of his quickſilver, diminiſhed by the longitudinal expanſion of the ſcale applied to the barometer, by which the height of the column was meaſured. The veſſel which contained BOERHAAVE's quickſilver was *glaſs*. The ſcale of DE LUC's barometer was *deal*. Glaſs is conſiderably altered in its dimenſions by heat; deal but very little, in the direction of its longitudinal fibres: therefore, though M. DE LUC neglected the expanſion of his ſcale, the expanſion which he obſerved of the quickſilver would be nearly the whole; but BOERHAAVE muſt have reckoned it much too ſmall, neglecting, as he ſays he did, the expanſion of his glaſs (o); and the difference is no more than what the expanſion of the glaſs will very nearly reconcile. BOERHAAVE's expanſion is too ſmall, to be in proportion to M. DE LUC's, by about $\frac{1}{288}$ of the whole volume, or ſomewhat more than $\frac{1}{300}$. The expanſion of glaſs, in length, by 180°, is, as I am informed, about $\frac{1}{100}$ of an inch upon a foot, or $\frac{1}{1200}$ part; therefore, its expanſion in ſolid content, or volume, will be very nearly $\frac{1}{400}$, by 180°; and, by 212°, about $\frac{1}{331}$, or leſs than $\frac{1}{300}$.

(o) *Sepoſitâ ergo vitri interea dilatatione*, &c. Theſe words may ſeem ambiguous; but the meaning ſeems ſufficiently determined by what is ſaid, p. 173. l. 2—5.

SECTION FOURTH.

Of the EQUATION for the TEMPERATURE of the AIR.

IF the temperature of the atmoſphere hath been any other than is expreſſed by + 16¼ of M. DE LUC's ſcale, the reſult of the calculation, formed upon the preceeding rules, requires a further correction. This correction ariſes from *a variation of the length of the ſubtangent of the atmoſpherical logarithmic*; which, as hath been already remarked, is found to be not conſtantly the ſame. That this matter may be the better underſtood, it will be proper to ſtate, in this place, what this ſubtangent is, in the nature of things, and upon what phyſical circumſtances its length depends.

Imagine then, that inſtead of the atmoſphere, in its natural ſtate, the earth were ſurrounded with an inelaſtic fluid, of an uniform denſity throughout, equal to that of the natural atmoſphere, in its loweſt parts, which are contiguous to the ſurface of the earth; imagine alſo, that every atom of this homogenous fluid were urged towards the earth's center, with an accelerative force, equal to that of gravity, at the ſurface of the earth. Now the preſſure of the atmoſphere upon the earth's ſurface, or on any given part thereof, being, at all times, a finite, though not a conſtant force, of which the phænomena of the Torricellian tube are a ſufficient proof, it is evident that it might, at any time, be equalled by the preſſure of a finite quantity of this imaginary fluid; and that, to render the preſſure of this fictitious atmoſphere, upon the whole, or any part of the earth's ſurface, equal to that of the real atmoſphere upon the like part, it would be requiſite to aſſign to it ſome finite thickneſs or depth. *Now* *that particular thickneſs or depth, of the imaginary fluid, which is, at any time, neceſſary to render an entire column of it a counterpoiſe for an entire column of the natural air, is, at that time, the length of the ſubtangent of the atmoſpherical logarithmic.* In ſhort, the ſubtangent of the atmoſpherical logiſtic, is the length of a column of ſuch a fluid as I have ſuppoſed, which would be ſuſtained in the Torricellian tube, by the preſſure of the air, at the level of the ſea, if we could ſuppoſe a tube of a ſufficient length.

This is demonſtrated by Mr. COTES, Harmon. Menſ. p. 18. and by no one elſe, that I know of, with equal ſimplicity.

It is a manifeſt conſequence from this, that the ſubtangent muſt always be as the preſſure of the whole cylindrical column (upon a given part of the earth's ſurface) directly, and the denſity of the air, at the ſurface, inverſely; and it may therefore appear to be repugnant to the theory already eſtabliſhed, to ſuppoſe it ſubject to variation. For if the preſſure be always as the denſity, upon which hypotheſis the whole theory is founded, that which is always as the preſſure directly, and the denſity inverſely, can be no other than a conſtant quantity. But M. DE LUC's experiments prove, beyond a doubt, that the atmoſpherical ſubtangent is variable; therefore the denſity of the atmoſphere, at the ſurface of the earth, at different times, is not proportional to the whole preſſure, at ſuch times, reſpectively. And in this there is nothing inconſiſtent with the foregoing theory, rightly underſtood. When the denſity of the air is ſaid to be as the compreſſing force, this is to be underſtood of air in the *ſame ſtate of elaſticity*; that is, in which the abſolute force of elaſticity, under all the different degrees of preſſure, is conſtantly the ſame. (The abſolute force of elaſticity is meaſured by the force exerted between two particles of given magnitudes and figures, at a given diſtance.) But there is no reaſon, from any experiments, to conclude, that the denſity of the air will be ſimply as the compreſſing force, in different abſolute forces of elaſticity. On the contrary, ſince the proportion of the compreſſing forces is found to obtain, between the denſities, when the abſolute elaſticity is conſtant; this alone is a proof, that the like proportion will not obtain, if the abſolute elaſticity be changed as well as the compreſſive force.

Imagine two parcels, ſimilarly ſhaped, A and B, of different fluids, the ſame in all their other properties, and ſimilarly, but unequally, elaſtic; that is, imagine the integrant particles of A to be equal in quantity of matter and bulk, and ſimilar in figure, to the integrant particles of B, ſeverally; and if each of the integrant particles of A, be itſelf an aggregate of leſſer integrants, each of the integrant particles of B is to be underſtood to be a like aggregate of equal and ſimilar leſſer integrants, ſimilarly compoſed; and whatever forces, except that of elaſticity, are exerted between the particles of A, imagine equal forces, of the ſame kinds, exerted according to the ſame laws, between the particles of B. Likewiſe, imagine the equal and ſimilar particles to be ſimilarly ſituated, in their reſpective maſſes. Such fluids are the ſame in all reſpects, elaſticity excepted. Further, imagine an elaſtic force to be exerted between every two integrant particles of A, and to vary, with the diſtance, as any power or function of the diſtance whatever. Imagine an elaſtic force, varying according to the ſame power or function of the diſtance, exerted between every two particles of B. Thus the fluids are *ſimilarly* elaſtic; but they are to be ſuppoſed *unequally* ſo; that is, the force exerted between two particles of the one, at any given diſtance, is to be greater than the force between two particles of the other, at the ſame diſtance. Now, if the law by which the elaſtic force varies, at different diſtances, in one of theſe fluids, as A, be ſuch, that the denſities of A, under different compreſſions, are to each other, as the compreſſive forces; the denſities of B, compared with each other, under different compreſſions, will likewiſe be as the compreſſive forces. But the proportion of the compreſſive forces will not ſubſiſt between the denſities of the two *different* fluids. The denſities will not be the ſame, when the compreſſive forces upon A and B are equal; nor will they be *as* the compreſſive forces, when thoſe forces are unequal. The denſities will be as the compreſſive forces directly, and the abſolute elaſticities inverſely; that is, the compreſſive forces will be as the denſities and abſolute elaſticities jointly, which is eaſily demonſtrated.

The compreſſive force upon any fluid, is to be eſtimated by the quantity of the preſſure, acting perpendicularly upon a given ſurface, or plane ſection of it. Whatever degree of preſſure acts upon the maſs of an elaſtic fluid, its particles will approach or recede, till the diſtances of every two adjacent particles are ſuch, that the whole elaſtic force, exerted perpendicularly againſt every ſurface or plane ſection of it, is preciſely equal to the preſſure acting perpendicularly upon the ſame ſurface, in an oppoſite direction. And in the particular degree of denſity, determined by that diſtance of the adjacent particles, the fluid will remain, while the ſame degree of preſſure is continued. Imagine therefore the denſities of the two maſſes, A and B, to be the ſame; then, from the ſuppoſed ſimilarity of the fluids, it follows, that the number of particles, exerting their elaſticities upon any equal and ſimilar ſections of A and B, muſt be equal; and that the diſtances of the correſponding particles, from each other, in the two maſſes, muſt be the ſame: conſequently the whole elaſticities, exerted upon equal and ſimilar ſections of the two maſſes, will be as the abſolute elaſticities of which they are compoſed. Therefore the compreſſive forces are, in this caſe, as the abſolute elaſticities. Call the common denſity of the fluids D; the compreſſive force upon A, P; upon B, Π; the abſolute elaſticities a, b, reſpectively. Let d denote ſome other denſity of the maſs A, and p the compreſſive force correſponding to that denſity.

Now D and d are different denſities of the ſame fluid A, under different compreſſive forces P, p.

Therefore, $p : P = d : D$ (by hypoth.)

But $P : \Pi = a : b$. (as hath been proved.)

Therefore, $p : \Pi = d \times a : D \times b$; that is, the compreſſive forces upon the different fluids A and B, when the denſities are unequal, are as the denſities and abſolute elaſticities jointly. Q. E. D. This demonſtration is independent of any more particular hypotheſis, concerning the law of the elaſticity, than barely that it is the ſame in both maſſes; and ſuch, in both, as to make the denſities of either always proportional to the forces by which it is compreſſed. But to this condition, a particular law of elaſticity is requiſite; namely, that the force exerted between every two adjacent particles diminiſh as the diſtance between them is increaſed; and, adopting this law, the compreſſive forces might be proved to be in the proportion aſſigned, that of the denſities and abſolute elaſticities jointly, by the ſame kind of reaſoning as is uſed to demonſtrate the 23d propoſition of the ſecond book of the *Principia*, which is only the moſt ſimple caſe of the more general theorem now propoſed.

Hence it follows, that if any cauſes act upon the atmoſphere, to change the degree of its abſolute elaſticity, provided they act in ſuch manner, as to change it equally at all heights; ſo that, though different at different times, it ſhall always, at any one time, be the ſame at all different heights; then, the denſities in all different parts of the cylindrical column, reſting perpendicularly upon a given ſmall part of the earth's ſurface, will, at any one time, be as the compreſſive forces upon each part reſpectively. But the denſities, in any one part of this column, at different times, when the abſolute elaſticities are different, will not be as the compreſſive forces upon that part, at ſuch *different* times reſpectively. Hence the relation between the decrement of denſity, and the increment of height, reduced according to the conſtruction of ſection ſecond, will, at every time, be repreſented, by the decrement of the ordinates, and the increment of the aſymptote of *ſome* logarithmic curve. But of *different* curves, at different times, or in different ſtates of the abſolute elaſticity; that is, the length of the ſubtangent of the curve, deſcribed in ſection ſecond, will vary: for the difference of the ſubtangents is the only thing that conſtitutes a real difference in two logarithmic curves. If ſuch curves have equal ſubtangents, though they have unequal ordinates, they are only *different parts* of the *ſame* curve.

The ſubtangent of the atmoſpherical logiſtic muſt always be as the preſſure of the whole column of the atmoſphere directly, and the denſity, at the earth's ſurface, inverſely; therefore it is directly as the abſolute elaſtic force. For call the ſubtangent S; the preſſure at earth's ſurface, P; the denſity, D; the abſolute elaſticity, A.

Now that S is as $\frac{P}{D}$ is obvious (from p. 245.); but $\frac{P}{D}$ is as A (by what hath now been proved). Therefore S is as A.

Now *heat* (p) is one cauſe, which is well known to influence the abſolute elaſtic force. An increaſe of heat increaſes elaſticity; and elaſticity is diminiſhed by a diminution of heat. Accordingly M. DE LUC's experiments ſhew, that when the temperature of the air is uniform (or the ſame at all heights) the ſubtangent of the atmoſpherical curve is increaſed, or diminiſhed, exactly in proportion to the increment or decrement of the uniform temperature, as indicated by the mercurial thermometer. His concluſion, from repeated experiments on the mountains near Geneva, is, *that, if* L *denote the difference of the tabular logarithms of the heights of the quickſilver, at two ſtations, corrected for the difference of the temperature of the quickſilver,*

(p) BOERHAAVE, Elementa Chemiæ, vol. i. p. 456, &c.

ſilver, and n *denote the difference of the temperature of the air, in degrees of his own ſcale, above or below* + 16¾, *then* $L \pm \frac{n}{215} L$, *is the difference of elevation in* 1000*ths of a Paris toiſe. The correction* $\frac{n}{215} L$, *to be added or ſubtracted, according as the temperature of the air is above or below* 16¾; *or according as* n *is a difference in exceſs or defect* (q).

But in any given value of *n*, the proportion of $\frac{n}{215} L$ to L is given, being that of *n* to 215. Therefore, $L \pm \frac{n}{215} L$ is the logarithm of the *ratio* of the heights of the barometer, in a ſyſtem of logarithms, in which the ſubtangent of the curve is to ſo many 1000ths of a Paris toiſe, as are expreſſed by the ſubtangent of the Briggian ſyſtem, as 215 ± *n* to 215. And this being the caſe, whatever be the value of L, in every given value of *n*, it is evident, that the variation of the *modulus* of the atmoſpherical ſyſtem, or of the ſubtangent of the atmoſpherical curve, is always as *n*, the variation of temperature.

But a uniform temperature is a condition of the atmoſphere, which rarely obtains, within ſmall diſtances, at leaſt, above the earth's ſurface; therefore the more uſual caſe of unequal temperatures muſt be conſidered.

When different temperatures obtain, at different heights, at the ſame time, they muſt render the abſolute elaſticities, at thoſe heights, unequal. Thus the hypotheſis of M. BOUGUER will take place;

(q) Recherch. ſur les Modif. de l'Atm. §. 607—611.

who, from a great number of experiments, made upon the CORDILEROS, and at various other heights above the level of the ſea, concluded, that the abſolute elaſticity of the air, which he calls, "the intenſity of the elaſtic force," muſt be different at different elevations. His conjectures indeed about the cauſe from whence the difference might ariſe, are not the moſt natural; and in one point he was evidently miſtaken: namely, that he imagined the abſolute elaſticity to be conſtant at every *given* elevation; and accordingly he hath traced the curve, which, according to his experiments, exhibits the laws of its variations from one height to another. This, it muſt be confeſſed, was truſting his expements too far, which were not made with the moſt accurate inſtruments; and this may have given occaſion to his learned countryman M. DE LA LANDE, to ſuppoſe that this curve may rather exhibit the deviations of his experiments from the truth, than any thing which really obtains in nature (r). His general hypotheſis, however, of a difference of abſolute elaſticity, in different parts of the atmoſphere, *muſt* obtain, whenever the temperatures of ſuch different parts are unequal; unleſs the effect of the inequality of temperature were to be compenſated by the ſynchronous unequal operations of ſome other cauſe. That it does in fact obtain, when the temperature is unequal, is proved, beyond a doubt, inſtead of being refuted, by M. DE LUC's experiments; and this is to be conſidered, as a further inſtance of their exact agreement with the genuine concluſions of accurate theory.

(r) Connoiſſance pour l'année 1765, p. 215.

When the abſolute elaſticities are different at different heights, the denſities will no longer be proportional to the preſſures; and the change of denſity, throughout the whole column of the atmoſphere, will no longer be repreſented by any *one* logarithmic. But to ſmall diſtances on one ſide or the other of different heights, it may ſtill be nearly repreſented by parts of *different* logarithmics. Now this is really the caſe, according to M. DE LUC: for, when the temperature, at two different heights, hath been different, he finds, *that the difference of thoſe heights will ſtill be expreſſed in* 1000*ths of a Paris toiſe, by the preceeding* formula, *viz.* $L \pm \frac{n}{215} L$, *if* n *be underſtood to denote the difference between the conſtant temperature* + 16¾, *and that which is the mean of the different temperatures of the two places of obſervation* (s); that is to ſay, though the temperature of that portion of the column of the atmoſphere, which is intercepted between the level of the two places of obſervation, hath not been the ſame, perhaps, in any two different parts; yet the variation of denſity and height will be exhibited, through the whole of this ſmall ſpace, without ſenſible error, by the curve, which would have repreſented them ſtrictly, if the entire column had been of the mean temperature of the extremes of this portion of it; namely, by the logarithmic, whoſe ſubtangent is expreſſed in 1000ths of a Paris toiſe, by $B \pm \frac{n}{215} B$, B denoting the ſubtangent of the Briggian ſyſtem. Imagine, therefore, that the barometer and thermometer have been obſerved at the ſame time, at three different heights. Put the difference of the tabular logarithms of the heights of the quickſilver, at the 1ſt and 2d elevation L; at the 2d and 3d, L″. Imagine the thermometer at the 1ſt, or loweſt height, to have been + $16\frac{3}{4}$; at the 2d, $16\frac{3}{4} + 2n$; at the third, $16\frac{3}{4} - 2m$.

(s) Recherch. ſur les Modif. de l'Atm. §. 663.

Then the diff. between 1ſt and 2d elevation, $= L + \frac{n}{215} L$

between 2d and 3d, $= L'' + \frac{n-m}{215} L''$

that is, the variations of height and denſity between each two of theſe three places, are nearly exhibited by two different logarithmics, the ſubtangents of which are $B + \frac{n}{215} B$; $B + \frac{n-m}{215} B$; therefore, the variations of height and denſity, throughout the atmoſphere, are not repreſented by any *one* logiſtic, when the temperature is unequal (as they would be, if, notwithſtanding an inequality of temperature, the abſolute elaſticities were the ſame in all parts); but by parts of different logiſtics, at different heights; as they ſhould be when the abſolute elaſticities are different, in different parts, and can only be conſidered as uniform to a ſmall diſtance, above or below any height aſſigned.

The mention of M. BOUGUER occurred ſo naturally upon this occaſion, that I muſt have reproached myſelf with an impiety towards the aſhes of a man, whoſe memory will ever be dear to ſcience, had I not attempted to vindicate his conjectures upon the point under immediate conſideration, from an animadverſion thrown out, in very general terms, with little tenderneſs, and only juſt in part. At the ſame time, I have pointed out, in what his principal miſtake conſiſts; which was, as M. DE LUC well obſerves, that, inſtead of ſeeking a rule which ſhould be general for all heights, and vary at every height, in proportion to the temperature, he ſet himſelf to find one which ſhould be general for all temperatures, at a particular elevation. The conſequence likewiſe, which he would have deduced from the unequal elaſticity of the particles of the air, that the leaſt elaſtic would be driven to the bottom, was undoubtedly erroneous. The exceſs of temperature may fall ſometimes in the lower parts of the atmoſphere, and ſometimes at greater heights; and where the greater temperature is, there, *cæteris paribus*, the elaſticity will be greater. The argument, which M. DE LUC directs againſt the exiſtence of ſenſible inequalities of elaſtic force in aggregate maſſes of the atmoſphere, derived from the ſuppoſed effect of the winds, throughout all the regions expoſed to their agitations(t), militates only againſt the probability of *permanent* inequalities in *given* places, ariſing from ſuppoſed ſpecific differences in the original conſtitution of the particles of the air; not againſt ſuch temporary inequalities, as we aſcribe to the occaſional energy of extraneous cauſes. An inequality of temperature undoubtedly exiſts, in aggregate maſſes, more frequently than the oppoſite. And from an inequality of temperature, whether in the aggregate or the diſcrete, neceſſarily follows, for the time, an analogous inequality of abſolute elaſtic force.

(t) Recherch. ſur les Modif. de l'Atm. §. 328.

Having now ſufficiently explained, what the correction is, for a variation of the temperature of the air, and whence it ariſes, I proceed to reduce M. DE LUC's *formula* to BIRD's Fahrenheit, and a ſcale of Engliſh fathom.

SECTION FIFTH.

M. DE LUC's RULES reduced to ENGLISH SCALES.

THE whole of this reduction I divide into three problems.

PROBLEM FIRST.

To find the length of the ſubtangent of the atmoſpherical curve, in thouſandths of a Paris toiſe, *the mean temperature of the air being given in degrees of* BIRD's *Fahrenheit.*

BY the compariſon of M. DE LUC's ſcale with BIRD's Fahrenheit, it appears, that $+ 16\frac{3}{4}$ of the former correſponds to $+ 69{,}25$ of the latter. Hence 69,25 is the temperature in BIRD's Fahrenheit, in which, the ſubtangent, of the atmoſpherical curve, is equal to ſo many 1000ths of a Paris toiſe, as are expreſſed by B, the ſubtangent of the Briggian ſyſtem. But the atmoſpherical ſubtangent is increaſed or diminiſhed by $\frac{1}{215}$ of this quantity for every degree of M. DE LUC's ſcale, above or below this given temperature; and a degree of M. DE LUC's ſcale is to a degree of BIRD's Fahrenheit as 178 to 80. Therefore, the ſubtangent varies by $\frac{8}{3827}$ of the ſame quantity, for every degree of BIRD's Fahrenheit, above or below the given temperature. Hence, if *n* denote the difference of the temperature of the air, or, in the caſe of unequal temperatures, the difference of the mean of the temperatures of the two ſtations, above or below 69,25 in degrees of BIRD's Fahrenheit; then, $B \pm \frac{n \times 8}{3827} B$ is the length of the ſubtangent in 1000ths of a Paris toiſe: that is, the ſubtangent of the atmoſpherical curve, in the temperature 69,25 ± *n*, is to ſo many 1000ths of a Paris toiſe, as are expreſſed by the *modulus* of the Briggian ſyſtem, as $3827 \pm \overline{n \times 8}$ to 3827.

PROBLEM SECOND.

To determine the temperature, in which the length of the ſubtangent of the atmoſpherical curve is expreſſed in thouſandths of an Engliſh fathom, *by the ſubtangent of the Briggian ſyſtem.*

FROM the number 69,25 ſubtract the 8th part of the number, to which 3827 bears the proportion of 10000 to 617; that is, from 69,25 ſubtract 29,51; the remainder 39,74 expreſſes the required temperature, in degrees of BIRD's Fahrenheit.

For, let S, Σ repreſent the ſubtangents of the atmoſpherical curves in the temperatures 69,25 and 69,25 — 29,51, reſpectively.

Then S : Σ = 3827 : 3827 — $\overline{29{,}51 \times 8}$ (by prob. 1.)

But 3827 : 3827 — 29,51 × 8 = 10000 : 10000 — 617 = 10000 : 9383.

Therefore, S : Σ = 10000 : 9383; that is, as one Paris toiſe to one Engliſh fathom. Therefore, whatever multiple S is of the Paris toiſe, or any part thereof, the ſame multiple is Σ of the Engliſh fathom, or its like part. And the length of S is expreſſed in 1000ths of a Paris toiſe, by the number which is the *modulus* of the Briggian ſyſtem (by prob. 1.); therefore, the length of Σ is expreſſed by the ſame number, in 1000ths of an Engliſh fathom.

PROBLEM THIRD.

To find the equation for every degree of BIRD's *Fahrenheit in the mean temperature of the air, above or below* 39,74.

CALL the variation of the length of the ſubtangent, correſponding to an increment or decrement of one degree of BIRD's Fahrenheit, V; and let S, Σ, as before, repreſent the ſubtangents correſponding to the temperatures 69,25 and 39,74.

Now V : S = 8 : 3827 (by prob. 1.)

And S : Σ = (3827 : 3827 − $\overline{29{,}51 \times 8}$ =) 3827 : 3591 very nearly (by prob. 1.)

Therefore, V : Σ = 8 : 3591.

that is, $V = \frac{8}{3591}\Sigma$. $(= \frac{1}{449}\Sigma$ nearly).

Hence the length of the ſubtangent of the atmoſpherical curve, in any temperature, 39,74 ± *n* is to its length in the temperature 39,74 as 3591 ± $\overline{8 \times n}$ to 3591; that is, putting B for the ſubtangent of the

the Briggian curve, B $\pm \frac{n \times 8}{3591}$ B is the length of the ſubtangent of the atmoſpherical curve, in 1000ths of an Engliſh fathom. And putting L for the difference of the tabular logarithms of the obſerved heights of the barometer, at two ſtations, corrected by the equation for the temperature of the quickſilver, and n for the mean difference of the temperatures of the air, in degrees of BIRD's Fahrenheit, above or below 39,74, the difference of the elevation of the two ſtations, is expreſſed in 1000ths of an Engliſh fathom by L $\pm \frac{n \times 8}{3591}$ L.

Upon theſe principles I have made a table, by which the equation $\frac{n \times 8}{3591}$ L may be computed for any temperature not above + 80, nor below 0 of BIRD's Fahrenheit, and for any height leſs than 10000 fathom.

It hath already been remarked, that the temperature of the quickſilver, in the portable barometer, may happen to be very different from that of the air, at the place and time of obſervation. For this reaſon, M. DE LUC adviſes, that every portable barometer ſhould be furniſhed with two thermometers; one fixed to the frame of the barometer, to indicate the temperature of the quickſilver; another, to be expoſed, at the time of obſervation, to the open air. The uſe of the annexed tables might be anſwered by a particular diviſion of the ſcales of theſe thermometers. The thermometers which accompany the two barometers, made for the ſociety, by Mr. EDWARD NAIRNE, under my directions,

are

are BIRD's Fahrenheits; but, to the fixed thermometer, I have had a particular ſcale applied, in which the fundamental interval, between melting ice and BIRD's boiling point, is divided into 81 equal parts. The equation for the temperature of the quickſilver is one fathom for every degree, of this ſcale, in the difference of temperatures. The thermometers, for the temperature of the air, have each a ſcale, in which the fundamental interval between melting ice and BIRD's boiling point, is divided into 120 equal parts. The point of 0 is placed at the 5th of theſe diviſions above melting ice. If n be the mean height of the thermometer, in degrees of this ſcale, at the two ſtations, the equation for the temperature of the air is $\frac{n}{300}$ L. + or —, according as n is poſitive or negative.

The place of the point of 0 upon the former ſcale is indifferent. It was put very low, that the temperatures of the quickſilver, at both ſtations, might always be above it. The computation, however, is rendered eaſier by the tables now given, than it can be by any ſuch peculiar diviſions of the ſcale.

It is to be particularly obſerved, that M. DE LUC always expoſed the thermometer, by which he meaſured the temperature of the air, *to the ſun*, if it happened to ſhine; but then the ball of his thermometer was always quite detached from the frame, which is a neceſſary precaution in this manner of uſing it. If the temperature of the air were meaſured by a thermometer, expoſed in the ſhade, I cannot but think the quantity of the equation would be different.

His reaſon for not ſhading his thermometer from the ſun, was this: when one ſide of the thermometer is expoſed to the ſun's rays, the other is in its own ſhade; therefore, in this ſituation, he thought it the juſt meaſure of the *mean* temperature of the air, which is neither the temperature of that part on which the ſun's rays fall, nor of that from which they are intercepted, but leſs than the one, and greater than the other, or a mean between the two(u). I confeſs, I ſhould have expected an irregularity, from heat excited by the rays of light, in their paſſage from the air into the glaſs, and from the glaſs into the quickſilver; and ſhould therefore have expoſed my thermometer in the ſhade; but the ſucceſs of M. DE LUC's experiments ſeems ſufficiently to juſtify the method he hath taken.

I ſhall cloſe this ſection with a brief ſolution of two phyſical problems, for which this ſeems the proper place.

PROBLEM FOURTH.

To compare the denſities of the air, at any given elevation above the ſurface of the earth, in different temperatures.

IF the barometer hath been obſerved, at any given elevation above the earth's ſurface, in different temperatures, reduce the obſerved heights of the quickſilver to a common temperature, if the temperatures of the quickſilver, as well as the air, have been different, at the different times of obſervation. Find the ſubtangent for each temperature of the air (by problem 3d); divide the corrected heights of the quickſilver by the ſubtangents correſponding to the obſerved temperatures of the air. The quotients are as the denſities in theſe temperatures reſpectively; that is, calling the heights of the mercury reduced, P, Π; the temperatures, T, Θ; the ſubtangents, S, Σ; and the denſities, D, Δ, $D : \Delta = \frac{P}{S} : \frac{\Pi}{\Sigma}$. For the denſities are as the compreſſive forces directly, and the abſolute elaſticities inverſely (by ſection 4th); and the abſolute elaſticities are as the ſubtangents (by ſection 4th); whence the truth of the rule is manifeſt.

(u) Recherch. ſur les Modif. de l'Atm. §. 533—536.

The moſt convenient method, however, for practice will be, to make the denſity of the air, in ſome given ſtate of the barometer and thermometer, a ſtandard, with which to compare the denſities in all other ſtates of theſe inſtruments. Suppoſe, for inſtance, at the level of the ſea, 30 inches be taken for the ſtandard height of the barometer, and + 40 for that of the thermometer. Let D be the denſity of the air, at the level of the ſea, when the barometer is 30 inches, and the thermometer + 40. Put P = 30 inches, and S = the ſubtangent of the logiſtic correſponding to the temperature + 40. Now in another temperature $40 + n$, let the ſubtangent be $S + \frac{b}{m} S$. And let the height of the barometer P be changed into $P + \frac{a}{r} P$; and let Δ be the denſity at the level of the ſea, in the temperature $40 + n$, and height of the barometer, $P + \frac{a}{r}P$. Then the difference between Δ and D, or $\Delta - D$, will be $D \times \frac{ma - br}{r \times m + b}$. Where obſerve, that b is poſitive or negative, according as n is poſitive or negative.

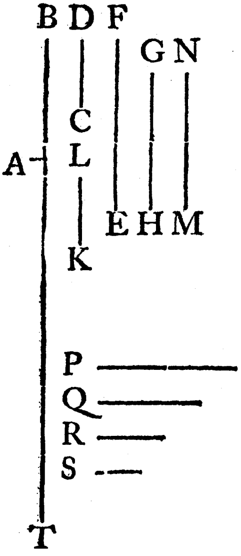
By this problem, the denſities of the air at a given height, in different temperatures, are to be compared with each other. Even good writers have hitherto been generally run away with by a notion, that the air's denſity, at a given elevation, at different times, would be always truly meaſured by the length of the mercurial column; than which nothing can be more erroneous, as hath been at large explained in the 4th ſection.

PROBLEM FIFTH.

The height of the quickſilver in the Torricellian tube, and the temperature of the air being given, at a given elevation, above the level of the ſea, to compare the denſity of the air with that of the quickſilver, at the time and place of obſervation.

LET TA repreſent the ſemi-diameter of the earth; T being the center, and A at the ſurface. Let B be a place at a given height, AB, above the ſurface. Let CD be the obſerved length of the quickſilver in the Torricellian tube, at the place B, in a given temperature of the air. It is required to

find



find the proportion between the denſity of the air, and that of the quickſilver, the obſerved temperature being ſuppoſed to obtain uniformly throughout the whole atmoſphere; or at leaſt, throughout a great portion of it, on all ſides of B.

Suppoſe it done; and that the denſity of the air, at the time and place of obſervation, is to the denſity of the quickſilver in the barometer, as the right line CD to EF. Now the temperature of the air being given, the ſubtangent of the logiſtic correſponding to the given temperature is given (by prob. 3d). Let GH be equal to that ſubtangent; and let KL be the perpendicular height of the mercurial column ſupported in the barometer at A, the earth's ſurface, at the time of obſervation at B. Now ſince EF is to CD as the denſity of quickſilver, in the barometer, to the denſity of the air at B, and CD is the perpendicular height of the column of quickſilver ſuſtained in the Torricellian tube at the place B, at the time of obſervation, it is evident, that EF is the perpendicular height of a column of an inelaſtic fluid, of the ſame denſity with the air at B, which, if it were urged with an accelerative force through its whole length every where equal to that of gravity at B, would, by its compreſſive force, ſuſtain the mercurial column CD: for the accelerative forces ſuppoſed to act upon the fluids in the columns EF, CD, being equal, and the heights of the columns being, reciprocally, as the denſities of the fluids, the compreſſive forces of the columns will be equal. But GH is the perpendicular height of a column of an inelaſtic fluid, of the ſame denſity with the air at A, the level of the ſea, which, if the whole were urged with an accelerative force equal to that of gravity at A, would ſuſtain the mercurial column KL (by ſect. 4). Suppoſe MN equal to the perpendicular height of a column of an inelaſtic fluid, of the ſame denſity with the air at B, which, if the whole were urged with an accelerative force, every where equal to that of gravity at A, would ſuſtain the mercurial column CD. It is evident that EF is to MN as the accelerative force of gravity at A to the accelerative force of gravity at B; for EF and MN are columns of fluids, of the ſame denſity, exerting equal compreſſive forces. Therefore, the heights of the columns muſt be reciprocally as the accelerative forces by which the compreſſive force is produced. But gravity at A is to gravity at B as the ſquare of TB to the ſquare of TA. Therefore, $EF : MN = TB^2 : TA^2$. Again, GH and MN are columns of fluids of different denſities, acted upon by equal accelerative forces; therefore, the compreſſive forces which they exert, that is, which they ſuſtain, will be as their heights and the denſities of the fluids jointly; that is, if P, Q be as the compreſſive forces of thoſe fluids, and R, S as their denſities, then $P : Q = GH \times R : MN \times S$. But the compreſſive forces ſuſtained by theſe columns are the preſſures of the air at A and B; alſo

the

the denſities of the fluids are the denſities of the air at A and B. Therefore the compreſſive forces are as the denſities; that is, $P:Q=R:S$. Therefore $GH\times R:MN\times S=R:S$. Therefore GH and MN are equal. Therefore $EF:GH=EF:MN$; but it hath been ſhewn, that $EF:MN=TB^2:TA^2$. Therefore $EF:GH=TB^2:TA^2$. But AB being given, TB is given, and the proportion of the ſquare of TB to the ſquare of TA is given. Therefore the proportion of EF to GH is given. And GH, is given as hath been ſhewn. Therefore EF is given, and the proportion of the given line EF to the given line CD, or of the denſity of the quickſilver to the denſity of the air, at B, is given. Q. E. I.

COMPOSITION.

Find the ſubtangent GH competent to the given temperature; find a line, EF, to which that ſubtangent ſhall bear the duplicate proportion of TA to TB; that is, of the earth's ſemi-diameter to the earth's ſemi-diameter increaſed by the given elevation of the place of obſervation, above the level of the ſea. As that line EF to the obſerved height of the quickſilver, ſo is the denſity of the quickſilver, in the barometer actually employed, to that of the air, at the time and place of obſervation.

Thus the ſpecific gravity of air may be found, comparing it with quickſilver, and by means of quickſilver with other fluids. Though it is only the moſt ſimple caſe of this problem that can ever come into practice, I choſe to diſcuſs it in its moſt general extent;

tent; as this is the only one method which gives entire ſatisfaction to the mind.

SECTION SIXTH.

I HAVE now gone through the expoſition and reduction of M. DE LUC's rules, for meaſuring heights by the barometer. The conſonance of them with theory appears, upon a ſtrict examination, to be ſuch, as ſtrongly confirms the principles, upon which the theory is founded. I ſhall conclude theſe diſquiſitions, with pointing out ſome further objects of enquiry, concerning the modifications of the atmoſphere, naturally ariſing out of M. DE LUC's diſcoveries, in conjunction with the theory already eſtabliſhed.

1. *It is probable, that the abſolute elaſticity of the air may be affected by various cauſes beſides heat.* The degree of humidity muſt occur to every one, as a circumſtance, which may reaſonably be ſuſpected to have ſome influence upon it; and, perhaps, the ſtate and quantity of the electricity of the air may have more.

2. *If M.* DE LUC's formulæ *are to be admitted as univerſally true, in all imaginable temperatures, there is a given temperature in which the elaſticity of the air would be deſtroyed, and, in any lower temperature, it would be negative; that is, the repulſion would be changed into attraction.* This given temperature is, —409,13 of BIRD's Fahrenheit; for if $B + \frac{n \times 8}{3591} B$, be the length of the

ſubtangent of the atmoſpherical curve, in 1000ths of an Engliſh fathom, whatever be the value of *n*, then, when $n = -448,875$, $B - \frac{8 \times n}{3591} B = 0$; that is, the ſubtangent vaniſhes; and the abſolute elaſtic force, which is always as the ſubtangent (by §. 4.) muſt vaniſh with it; but when $n = -448,875$ the temperature is $-409,13$.

Perhaps it may be thought more probable, that the variation of the ſubtangent, or of the elaſtic force, is not preciſely as the variation of temperature. If the ſubtangent changes in a *geometrical* proportion while the temperature, as ſhewn by the thermometer, changes arithmetically, the ſubtangent, or the abſolute elaſtic force, will not vaniſh with any aſſignable decrement of temperature; and in that temperature, in which it ſhould vaniſh, accords to M. DE LUC's *formulæ*, it will ſtill remain more than $\frac{3}{10}$ths of what it is in the temperature $+ 39,74$; and yet the equation, for an increment or decrement of temperature, amounting to 40°, will not differ from M. DE LUC's by more than four fathom in the height of 1000. I muſt repeat, that I am now only pointing out the concluſions of theory, as hints of further enquiry. I do not mean to ſubſtitute this hypotheſis as more accurate than M. DE LUC's in practice; I do not affirm, that it is more true in theory. I mean only to ſuggeſt, that if M. DE LUC's *formulæ* are admitted as mathematically true, a conſequence will follow, which may ſeem to ſome unlikely to obtain in nature. That however this conſequence, if otherwiſe improbable (which is not the opinion to which, for my own part, I incline) is not to be too haſtily adopted, upon the bare evidence of its ariſing out of M. DE LUC's *formulæ*; becauſe the general extent of theſe *formulæ* is more than experiment hath hitherto proved. And I ſhew, by a very ſimple inſtance, that it is eaſy to imagine other laws, between the temperature and the elaſticity, which, to all practical purpoſes, would be the ſame as M. DE LUC's, within the utmoſt limits of his experiments, and even beyond them, and yet differ from it in remote conſequences.

If M. DE LUC's *formulæ* be univerſally and ſtrictly true, the conſequence I have pointed out muſt be admitted; and it will follow, that the force of elaſticity owes its firſt production and origin to temperature: or, at leaſt, that the relation between heat and elaſticity, if not the moſt intimate one of cauſe and effect, is that which ſtands the next in the ſcale of natural affinities, namely, that they are inſeparable effects of ſome common cauſe. And theſe concluſions will hold, if the form of the general expreſſion be true, though the quantity of the equation $\frac{n \times 8}{3591}$ B ſhould require a correction: for a change of the fraction $\frac{8}{3591}$ will only alter the particular temperature, in which the ſubtangent and elaſticity vaniſh.

3. *The diminution of the air's denſity, as we aſcend from the ſurface, is ſubject to a limit.* This limit is different, in different heights of the barometer, at the level of the ſea, and even in a given height of the barometer, in various temperatures. But the denſity even at an infinite height is never 0, or infinitely ſmall. In fig. 1. through C draw

C draw CF, parallel to AD, meeting the atmoſpherical curve in F. CF is the denſity of the atmoſphere, at an infinite height (by p. 231). The proportion of CF to the denſity at the level of the ſea is given in any uniform given temperature of the air. For the uniform temperature being given, the ſubtangent of the curve DF is given; and conſequently, the proportion is given, of which the given line CA is the logarithm. The diminution of compreſſive force is ſubject to a correſponding limit.

This curious circumſtance hath been remarked by no one, that I know of, but Dr. BROOKE TAYLOR, whoſe writings are too little attended to, even among mathematicians, from an opinion which prevails of their obſcurity. That conſummate geometer ſeems indeed to have thought it improbable, that a finite denſity of the atmoſphere, at infinite diſtances, though the neceſſary conſequence of the theory, ſhould actually exiſt; and, for this reaſon, he would imagine ſuch a law of the elaſtic force, as ſhould render the denſity of the atmoſphere, beyond a certain height, much greater than in proportion to the compreſſive force; and circumſcribe the whole within narrow limits (w). And if the theory inferred a *great* denſity at infinite or even at great heights, ſuch a denſity as would ſenſibly diſturb the motions of the planets, it would be neceſſary to have recourſe to ſome ſuch hypotheſis; but, as this is not the caſe, I ſee no ſuch neceſſity. I know not, for what reaſon, mathematicians have been afraid to admit the infinitude of the atmoſphere of the earth; whether they thought it would bear hard upon the Newtonian doctrine of a void; or, that it implied the infinitude of matter. But neither the one nor the other of theſe conſequences is to be apprehended: for neither the phænomena of nature, nor the principles of the Newtonian philoſophy, require, that there ſhould be any where a great chaſm in the univerſe; or that the whole material world ſhould be actually circumſcribed within any finite ſpace. A large proportion of *pore*, or interſperſed vacuity, is ſufficient for all purpoſes. Nor doth an abſolute infinity of matter by any means follow from the hypotheſis of an infinite *number* of finite maſſes; and an infinite number of finite maſſes is all that is implied in the notion of a rare elaſtic fluid, diffuſed throughout infinite ſpace. I agree, indeed, with Mr. COTES, that there are no *data* from which any great altitudes of the atmoſphere can indubitably be concluded, in the way of experiment: but I do contend, that there are no *data*, from which the ſuppoſition of its infinite height can, in the ſame way, be diſproved. And this may juſtly be held more probable than the contrary, as being the conſequence of a theory which hath never yet, in any inſtance, proved fallacious; and this I venture to aſſert, with the leſs heſitation, as, beſides the evident reaſon of the thing, the great authority of NEWTON is on my ſide. The infinite extent of the earth's atmoſphere is manifeſtly ſuppoſed, in that ſubtle diſquiſition concerning the tails of comets, which occurs in the poſthumous work *De Syſtemate Mundi*. Inſtead of framing hypotheſes, therefore, to remove imaginary difficulties, I ſhall purſue the theory as far as it will lead; and my next ſtep ſhall be, to exhibit the ſucceſſive rarefactions of the atmoſphere, as we aſcend from the earth, in a table.

(w) Method. Increment. prop. 26. Scholium.

	Heights in miles	Volume.
0,000	00,0	1
0,01	40,0	3069
0,02	80,8	$\overline{3069}\rceil^{2}$
0,03	122,6	$\overline{3069}\rceil^{3}$
0,04	165,0	$\overline{3069}\rceil^{4}$
0,05	208,5	$\overline{3069}\rceil^{5}$
0,06	252,1	$\overline{3069}\rceil^{6}$
0,07	298,2	$\overline{3069}\rceil^{7}$
0,08	344,5	$\overline{3069}\rceil^{8}$
0,09	391,9	$\overline{3069}\rceil^{9}$
0,10	440,3	$\overline{3069}\rceil^{10}$
0,11	489,7	$\overline{3069}\rceil^{11}$
0,12	540,6	$\overline{3069}\rceil^{12}$
0,20	990,6	$\overline{3069}\rceil^{20}$
0,30	1698,1	$\overline{3069}\rceil^{30}$
0,40	2641,7	$\overline{3069}\rceil^{40}$
	Semidiameter.	
0,50	1,00	$\overline{3069}\rceil^{50}$
0,60	1,50	$\overline{3069}\rceil^{60}$
0,70	$2\frac{1}{3}$	$\overline{3069}\rceil^{70}$
0,80	4,00	$\overline{3069}\rceil^{80}$
0,90	9,00	$\overline{3069}\rceil^{90}$
0,98	49,00	$\overline{3069}\rceil^{98}$
0,9836 +	62,00	$\overline{3069}\rceil^{98,36+}$
1,00	Infinite.	$\overline{3069}\rceil^{100}$

This table ſhews the rarefactions of the atmoſphere at different heights, above the ſurface of the earth, upon a ſuppoſition, that the temperature, throughout the whole, is that which is indicated by + 40° of BIRD's Fahrenheit. The middle column ſhews the heights above the ſurface, correſponding (according to the conſtruction of the atmoſpherical logarithmic, §. 2) to the decimal parts of the earth's ſemi-diameter expreſſed by the numbers in the firſt column; and in the third the correſponding rarefactions, or the proportions of the ſpaces which a given quantity of air would occupy at the different heights, are expreſſed by the powers of the number 3069.

The principle and method of the conſtruction of this table is obvious. I compute the length of the mean ſemi-diameter of the earth in thouſandths of an Engliſh fathoms. I divide that number by 100. The quotient is the tabular logarithm of the 100th root of a number, which is to unity, as the denſity at the ſurface to the denſity at an infinite height, in the temperature ſuppoſed; or, which is the ſame thing, as the ſpace occupied by a given quantity of air, at an infinite height, to the ſpace occupied by the ſame quantity, at the ſurface. This root I call the *radical number*. Then, imagining the earth's ſemi-diameter, divided into 100 equal parts, and numbering thoſe parts 1, 2, 3, *&c*, downwards, towards the center, I compute the heights, above the ſurface, correſponding to thoſe parts ſucceſſively, according to the conſtruction of the atmoſpherical logarithmic, §. 2; and, writing the reſulting numbers in the ſecond column, in the third, I write the powers of the radical number, increaſing by unit, in regular ſucceſſion downwards.

The proportion in which the atmoſphere will be rarefied, at given heights above the ſurface, will be very different, in different temperatures. It may always be exhibited by a table of this form, but every different temperature will have its own radical number. The radical number, in the temperature + 40, is 3069; and, for any other given temperature, may be thus found. Call the tabular logarithm of 3069, y; and let n be the number of degrees of BIRD's Fahrenheit, in the difference between the given temperature, for which the radical number is to be ſound, and + 40. Then $y \pm \frac{n}{449 \mp n} y$ will be the tabular logarithm of the radical number, for the temperature aſſigned: obſerving, that n is to be poſitive or negative in the denominator of the coefficient $\frac{n}{449 \pm n}$, according as the

the given temperature is greater or leſs than + 40; and that the term $\frac{n}{449 \pm n} y$ is negative when n in the denominator of its coefficient is poſitive, and *vice verſa*. Thus the radical number, for the temperature 0, is 6731,2 +.

This table is not intended for any practical purpoſes; but merely for ſpeculative amuſement. They, who take delight in the contemplation of final cauſes, will remark, with admiration, how large a part of the whole rarefaction of the atmoſphere is performed on this ſide of the moon. Indeed there is comparatively but a very ſmall part performed beyond it: ſo that the moon revolves at a diſtance where the reſiſtance, from the earth's atmoſphere, is reduced almoſt to its *minimum*. For if we imagine a ſeries of quantities, conſiſting of 101 terms, decreaſing from the firſt continually in geometrical proportion; and ſuch, that the firſt and greateſt ſhall be to the laſt and leaſt, as the denſity of the atmoſphere, at the earth's ſurface, to the denſity at an infinite diſtance; the denſity, at the mean diſtance of the moon, will be leſs than the 99th of theſe, or the leaſt but two.

4. *An increaſe of temperature rarefies the lower regions of the atmoſphere, more in proportion than the upper, and brings the conſtitution of the whole nearer to that of an uniform denſity.* Though the effect of heat alone, upon every orb of the atmoſphere ſeparately, is to rarefy each, perhaps, proportionally, yet its operation upon the different orbs is ſo modified, by the different degres of preſſure

they ſuſtain, and by the communication between them, that the greater the temperature of the atmoſphere, though uniform throughout, the leſs will the proportion be of the denſity at any given height to the denſity at any greater given height. For the greater the temperature, the greater the ſubtangent; and the greater the ſubtangent, the leſs the proportion of AD to CF, of which the given line AC is the logarithm. (*Vid.* fig. 1.)

5. *If at any height above the ſurface of the earth a given alteration of temperature diminiſh the air's denſity in the ſame proportion, as it increaſes the abſolute elaſticity, or* vice verſâ, *the* preſſure *of the ſuperincumbent atmoſphere, at that height, will remain unchanged. At all lower heights, the preſſure will be leſs, than in a cooler condition of the atmoſphere, and greater at all greater heights. On the contrary, the preſſure at all lower heights will be greater than in a warmer condition, and at all greater heights leſs.* For let CA (fig. 2.) repreſent the ſemi-diameter of the earth, the curve DEF the atmoſpherical logarithmic for a certain temperature, and GHK the logarithmic for another greater temperature. Let the ordinates of the two curves AD, AG be as the denſities, at the earth's ſurface, in the different temperatures, to which the curves belong, reſpectively. Then it is evident, the ordinates βE, βH, drawn through any other point β in the aſymptote, will be as the denſities, at the height to which Aβ correſponds, in the different temperatures, reſpectively. Now, ſuppoſe that the denſity of the air, at the height B, in the greater temperature, is leſs than the denſity at

the ſame height in the cooler temperature, in the ſame proportion, as the abſolute elaſticity in the greater temperature exceeds the abſolute elaſticity in the leſs. Then I ſay firſt, that the preſſure of the ſuperincumbent atmoſphere at B, is the ſame in both temperatures. For take $C\beta = \frac{CA^2}{CB}$, and draw the ordinate βE, cutting the curves in E and H, and through E and H draw tangents to the curves, EL, HM, meeting the common aſymptote, AC, in L and M. Now the ſubtangents βL, βM, are as the abſolute elaſticities in the different temperatures (by p. 250.). And βE, βH are as the denſities at B (by conſtruction). Therefore $\beta H : \beta E = \beta L : \beta M$. Therefore $\beta H \times \beta M = \beta E \times \beta L$. But the rectangle $\beta E \times \beta L$ is equal to the area intercepted by the ordinate βE, the curve EF, and the aſymtote βC, infinitely produced. And the rectangle $\beta M \times \beta H$ is equal to the area intercepted by the ordinate βH, the curve HK, and the aſymptote βC infinitely produced. Therefore theſe areas are equal. And theſe areas are as the preſſures of all the atmoſphere above B, in the temperatures to which the curves belong, reſpectively. Therefore the preſſures in theſe different temperatures are equal. *Q. E. D.* I ſay further, that the preſſure of the ſuperincumbent atmoſphere, at any height below B, is leſs in the greater temperature than in the cooler. Let AP be any height leſs than B, and take $Cp = \frac{CA^2}{CP}$, and draw the ordinate pN, cutting the curves in the points N and O. Now pN, βE being as the denſities at P and B, in the cooler condition of the air, and pO, βH as the denſities at the ſame heights P and B, in the increaſed temperature, pO is leſs than to bear to βH, the proportion of pN to βE (by 4th of this): and, by permutation, pO is leſs than to bear to pN the proportion of βH to βE, or of βL to βM. Therefore $pO \times \beta M$ is leſs than $pN \times \beta L$; that is, the area intercepted between the ordinate pO, the curve OHK, and the aſymptote pC infinitely produced, is leſs than the area intercepted by the ordinate pN, the curve NEF, and the aſymptote pC infinitely produced. But again, theſe areas are as the preſſures of all the atmoſphere above P, in the temperatures to which the curves reſpectively belong; therefore, the preſſures in the greater temperature, to which the curve OHK belongs, is leſs than the preſſure in the cooler temperature, to which NEF belongs. *Q. E. D.* In like manner it may be ſhewn, that the preſſures in the warmer temperature, at all heights above B, are greater than in the cooler.

And thus theory might have brought us to expect a phænomenon, which M. DE LUC hath actually obſerved, and was not a little ſurpriſed at. For if the temperature of the atmoſphere be at any time gradually augmented, a barometer, placed below the height of ſtationary preſſure, will ſink, while another, placed higher up in the atmoſphere, will riſe. This is what M. DE LUC hath obſerved in two barometers at the foot and ſummit of a hill, in ſettled weather, while the natural heat of the day hath been upon the increaſe; from whence he, with reaſon, concludes, that between theſe two ſtations, where the preſſure was changing, at the ſame time, contrary ways, there muſt have been an intermediate one, which I call the height of ſtationary preſſure, where no change, in either ſenſe, could take place.

I ſhall hereafter ſhew, at what height the place of unaltered preſſure ſhould fall, by theory, for every change of temperature. It ſeems a problem worthy of a naturaliſt, to enquire how far theory doth, in this circumſtance, agree with the real operations of nature.

6. It may ſeem, perhaps, ſtill more ſurprizing, but it is no leſs true, that *there will generally be a particular height in the atmoſphere where the* denſity *will remain unchanged, by a given change of temperature.* To determine in what changes this will happen, and at what height the place of unaltered denſity, for given changes of temperature, ſhould fall, requires only the ſolution of the following problem.

PROBLEM FIRST.

To find the interſection of two logarithmics, which have a right line given in poſition for their common aſymptote, and their ſubtangents given in magnitude; an ordinate in each curve, drawn at right angles with the common aſymptote, through a given point in it, being alſo given in magnitude.

IMAGINE two logarithmics, CDE, FDG (fig. 3, 4, 5, 6.), having the right line AB, given in poſition, their common aſymptote; and ſuppoſe the ſubtangent of each curve given in magnitude. Through a given point A, in the common aſymptote, imagine the right line AC drawn, an ordinate at right angles with the aſymptote, meeting the curves in C and F; and let AC, AF be ſeverally given in magnitude. It is required to find the point where theſe curves interſect. Suppoſe it done, and let D be the interſection. Draw DL perpendicular to AB. Through F, the point where AC meets one of the curves FDG, draw FM parallel to AB, meeting the other curve CDE in M. Draw MN perpendicular to AB, and take AH, AK equal to the given ſubtangents of the curves, CDE, FDG, reſpectively. Now AL is the logarithm of the *ratio* of AF to LD, in the ſyſtem of the curve FDG; and (becauſe NM = AF) NL is the logarithm of the ſame *ratio*, in the ſyſtem of the curve CDE. Therefore, AL : LN = AK : AH. Therefore the proportion of AL to LN is given; and conſequently, that of AN to AL is given. But AN is given in magnitude. For AC and AF are given in magnitude (by hypotheſis). Therefore, the proportion of AC to AF or NM is given; and AN is the logarithm of that given proportion in the ſyſtem of the given curve CDE. But AN being given in magnitude, and the proportion of AN to AL being given, AL is given in magnitude. And it is given in poſition, and the point A is given (by hypotheſis). Therefore the point L is given (by 27. dat.). Therefore, LD, being perpendicular to AL, is given in poſition (by 30. dat.). But AL being given in magnitude, the proportion of AF to LD is given (by logarithms). And AF is given in magnitude. Therefore LD is given in magnitude. Therefore the point D is given. *Q. E. I.*

The conſtruction is obvious. It is evident, that the points L and N are on the ſame ſide of A, if F be at the greater curve, as in fig. 3 and 4; but on different ſides of A, if the curve to which F belongs be the leſs, as in fig. 5 and 6 (*x*).

The calculation of the lengths AL, LD, by means of the logarithmic canon, is very ſimple. Putting B for the ſubtangent of the Briggian ſyſtem, L for the tabular logarithm of AC, and D for the difference of the tabular logarithms of AC, AF, we ſhall have,

$$\text{Firſt, } AL = \frac{AH \times AK \times D}{B \times KH}.$$

$$\text{And again, } L \mp \frac{\overline{AK \times D}}{KH} = \text{tab. log. of } LD.$$

In this ſecond expreſſion, the ſecond term is negative, if the greater of the given ordinates belong to the leſs curve, as in fig. 3 and 5; but poſitive if the greater ordinate belong to the greater curve, as in fig. 4. and 6.

Both theſe theorems are ſo eaſily derived from the preceeding analyſis of the problem, that it is needleſs to add the ſynthetic demonſtration; but they may be reduced to more commodious forms for practice by the following artifice.

(*x*) By the greater and the leſs curve I mean that which hath the greater or the leſs ſubtangent.

Firſt, if AH be leſs than B, and AK greater, put AH $=B-\frac{p}{q}B$, and AK $=B+\frac{s}{t}B$. Then HK $=\overline{\frac{s}{t}+\frac{p}{q}}\,B$. And ſubſtituting theſe values for AH, AK, and HK, reſpectively, we ſhall have, by the theorem, for AL,

$$AL=\frac{qt+qs-tp-ps}{qs+tp}D.$$

And hence if $t=q$, $AL=\frac{q+s}{p+s}D\times\overline{1-\frac{p}{q}}$.

2. If AH and AK both exceed B, put AH $=B+\frac{p}{q}B$, and AK $=B+\frac{s}{t}B$. Then HK $=\overline{\frac{s}{t}-\frac{p}{q}}\,B$.

In this caſe $AL=\frac{qt+qs+tp+ps}{qs-tp}D$.

And if $t=q$, $AL=\frac{q+s}{s-p}D\times\overline{1+\frac{p}{q}}$.

3. Again, if both AH and AK be leſs than B, put AH $=B-\frac{p}{q}B$, and AK $=B-\frac{s}{t}B$. Then, HK $=\overline{\frac{p}{q}-\frac{t}{s}}\,B$. And ſubſtituting theſe values, $AL=\frac{qt-qs-tp+ps}{tp-qs}D$; or if $t=q$,

$$AL=\frac{q-s}{p-s}D\times\overline{1-\frac{p}{q}}.$$

And, by the theorem, for LD, we ſhall have by due ſubſtitution, in the firſt caſe,

$L\mp\frac{qs+qt}{qs+tp}D=$ tab. log. LD; or, if $t=q$,

$$L\mp\frac{s+q}{s+p}D=\text{tab. log. LD.}$$

In the 2d caſe, $L \mp \frac{qt+qs}{qs-tp} D = \text{tab. log. } LD$; or, if $t=q$,

$$L \mp \frac{s+q}{s-p} D = \text{tab. log. } LD.$$

In the 3d caſe, $L \mp \frac{\overline{t-s}\times q}{tp-qs} D = \text{tab. log. } LD$; or, if $t=q$,

$$L \mp \frac{q-s}{p-s} D = \text{tab. log. } LD.$$

The *formulæ*, in which q and t are equal, will be of particular uſe in the application of this theory to the atmoſphere, of which examples will ſhortly be given.

In Caſe 1, if $\frac{p}{q} = 0$; that is, if $AH = B$,

$$AL = D \times \overline{1 + \frac{t}{s}},$$

$$\text{and } L \mp D \times \overline{1 + \frac{t}{s}} = \text{tab. log. } LD.$$

$$\text{But if } \frac{s}{t} = 0,\ AL = D \times \overline{\frac{q}{p} - 1};$$

$$\text{and } L \mp D \times \frac{q}{p} = \text{tab. log. } LD.$$

COR. *The interſection of two logarithmics being given, which have a right line given in poſition for their common aſymptote, and ſubtangents ſeverally given in magnitude; to find the point in the common aſymptote, through which the ordinate, drawn at right angles with the aſymptote, is cut by the curves in a given proportion, and to aſſign the magnitude of each ſegment,* is the converſe of the foregoing problem; and the ſame principles lead to its ſolution. For ſuppoſe A the point required. The proportion of AC to AF being given (by hypotheſis), the length of AL will be given, from the ſame analyſis as before. But D being given, and the right line AB given in poſition, LD is given in poſition and magnitude; and, AL being given, the proportion of LD to AC, and alſo to AF, is given (by logarithms). Therefore AC and AF are each given.

The expreſſion for the length of AL is the ſame as before. And tab. log. LD $\pm \frac{AK \times D}{KH} = L$. In this expreſſion the ſecond term is poſitive, if the ordinate from A to the leſſer curve is to be the greater of the two; in the contrary caſe, negative.

Now imagine CDE, FDG to be logarithmics of the atmoſphere, in different temperatures; AC being the denſity at the earth's ſurface in one temperature, and AF in the other; and let AB be the ſemi-diameter of the earth: let the two logarithmics meet in D, and draw DL perpendicular to AB. Now if the point L be any where in the line AB, between A, which is at the ſurface of the earth, and B, which is the center, the ordinate LD will repreſent the denſity of the air, in the ſyſtem of both curves, at the diſtance $\frac{BA \times AL}{BA - AL}$ above the earth's ſurface; and therefore, at this height, the denſity is the ſame in the one temperature as the other.

If L coincide with B, DL repreſents the denſity at an infinite height; but if L falls beyonds B, DL is not among the ordinates, of either curve, which repreſent denſities any where exiſting. The expreſ-

ſion

ſion $\frac{BA \times AL}{BA - AL}$, which is infinite when $AL = AB$, now becomes more than infinite, the denominator being negative; and there is no height in the atmoſphere, at which the denſity is the ſame in both temperatures. Again, if L fall above the earth's ſurface (as in fig. 4. and 6.), LD is not among the ordinates of either curve which repreſent denſities; and AL, being negative, the expreſſion $\frac{BA \times AL}{BA - AL}$ becomes $-\frac{BA \times AL}{BA + AL}$; which expreſſes a diſtance below the earth's ſurface; but whether L fall above or below the ſurface depends upon the ſtate of the denſity at A, and the temperature jointly. If the denſity were to be greater, in the greater temperature, then the greater ordinate at A belongs to the greater curve (as in fig. 4. and 6.), and L is above the ſurface: but if the denſity be leſs, when the temperature is greater, then, of the two ordinates at A, the leſs belongs to the greater curve, and L is below the ſurface (as in fig. 3. and 5.); and in this caſe, the place of the point L depends upon the magnitude of AN, and the proportion of HK to AK. If $HK : AK = AN : AB$, then L and B (in fig. 3. and 5.) coincide, and the denſities are the ſame at an infinite height. If HK be leſs than to bear to AK, the proportion of AN to AB, AL will be greater than AB, and L will fall beyond B, and the denſities are no where the ſame. But if HK be greater than to bear to AK, the proportion of AN to AB, AL will be leſs than AB; and in this caſe thre will be a height above the earth's ſurface, namely, $\frac{BA \times AL}{BA - AL}$, at which the denſities, in both temperatures, will be the ſame. And it is evident, that this height is given, if AL be given. But AL is given, by the preceeding problem, if the ſubtangents AH, AK be given in magnitude, and the proportion of AC to AF be given. But if each temperature be given, each ſubtangent is given (by ſect. 5. prob. 3.) and the proportion of AC to AF will be given by the barometer. (ſect. 5. prob. 4.)

EXAMPLE.

1744,	d	h	B^{rs} obſerved.	T^{r} in.	T^{r} out.	B^{rs} reduced to common temperature	Denſities, by §. 5. prob. 4.
March	26	1½	30,17	54	61	30,176	137904
April	2	6	29,37	56	48	29,37	138039

$$\text{Hence } D = 0.0004080 = 4{,}08 \text{ fathom.}$$

But $p = 8.$ $s = 21.$ $q = 449.$ $q + s = 470.$ $s - p = 13.$

$$\text{and } \frac{q+s}{s-p} = 36{,}1.$$

Therefore by *formula* 2d, AL = 150 fathom; and at a height, inſenſibly greater, the denſity was the ſame in both conſtitutions of the atmoſphere.

7. *It is manifeſt, that the changes of denſity above and below the height where it remains unaltered, are contrary. If the lower denſities are diminiſhed, the higher ones are increaſed, and* vice verſâ. Notwithſtanding the mathematical evidence of theſe concluſions, I am perſuaded, it will appear to many a *phyſical*

paradox, that heat ſhould any where *condenſe.* I have already hinted at the ſolution. Heat doth not condenſe any finite maſs of matter, which is at liberty, on all ſides, to expand itſelf; but acting on a finite maſs, which hath not unlimited liberty of expanſion, it may condenſe one part by the rarefaction of another. Acting on an infinite maſs, it muſt do this; becauſe it can only tranſpoſe. It neither generates new matter, nor annihilates what already is. What is taken therefore from one part, muſt be added to another, and *vice verſâ*; otherwiſe the quantity of matter, in the whole, muſt be changed.

Imagine ABCD to be a ſmall portion of any orb of the atmoſphere (fig. 8.) AEFB, CMND, CHGA, DLBK, contiguous portions. Heat drives many of the particles, which occupy the ſpace ABCD, out of it; but it likewiſe drives out many of the particles which occupy the contiguous ſpaces. And of thoſe which are driven out of the contiguous ſpaces, many will enter the ſpace ABCD. If the particles which are driven *into* ABCD, be more in number than thoſe which are driven *out* of it, the air, in this ſpace, is condenſed, by that very cauſe, which would rarefy it, if the contiguous portions were annihilated. Thus condenſation in one part may, in an infinite fluid, muſt enſue from rarefaction in another, if the quantity of matter remains unaltered. Where then is the wonder, that the like effect ſhould follow from the cauſe of rarefaction combined with other cauſes?

8. *The caſes in which* L *falls below* B, *or above the ſurface, ſeem to be phyſically impoſſible, without*

an alteration of the quantity of matter in the atmoſphere; for without this, it cannot be every where rarefied, or every where condenſed at one and the ſame time.

9. *Having found the heights where the denſity is the ſame in any two different temperatures, the height where the preſſures in different temperatures are equal, will eaſily be determined.* For this purpoſe, we have only to ſeek the ſolution of the following problem.

PROBLEM II.

Two logarithmics (DQF, GQK) *interſecting in a given point* (Q), (*Vid. fig.* 2.), *having a right line* (AC) *given in poſition for their common aſymptote, and ſubtangents ſeverally given in magnitude; to find the point* (β) *in the aſymptote, where an ordinate* (βHE) *being drawn at right angles, to meet both curves, the areas intercepted between the two curves and the common aſymptote infinitely extended, beyond the ordinate* βE, *are equal.*

LET βL, βM be equal to the ſubtangents of the curves DQF, GQK, reſpectively. Becauſe the curvilinear areas are equal; therefore, the rectangles $\beta L \times \beta E$, $\beta H \times \beta M$, are equal. Therefore, $\beta E : \beta H = \beta M : \beta L$. But βM, βL, being given, the proportion of βM to βL is given. Therefore, the proportion of βE to βH is given. Therefore, the point β is given (by corollary of preceeding problem).

CONSTRUCTION.

Find the point β (by corollary of preceeding preproblem) ſuch that the ordinate βHE being drawn, βE, βH, may be reciprocally as the ſubtangents; or that βE may be to βH as the ſubtangent of the curve GHK to that of DEF: and the thing propoſed is done.

Now imagine the curves GQK, DQF, to repreſent the logarithmics correſponding to different given temperatures of the air, A being at the ſurface of the earth, and C at the center. And having found the point β, take $AB = \frac{A\beta \times CA}{CA - A\beta}$; then B is the place where the barometer will ſtand at the ſame height, in both conditions of the atmoſphere.

LIMITS.

Draw the ordinates, AGD, CFK, and QR. If the point Q be any where below the ordinate through A, it is evident, the point β will lye above the point R. And if the ſubtangents βL, βM, be reciprocally as the denſities AD, AG, the points β, A, and conſequently B, A, coincide, and the preſſure, at the level of the ſea, remains unchanged.

If the greater ſubtangent βM be greater than to bear to the leſs βL, the proportion of the greater denſity AD to the leſs AG, β will fall above A, and B conſequently will be below it; for Aβ becoming negative, the expreſſion $\frac{CA \times A\beta}{CA - A\beta}$ becomes $-\frac{CA \times A\beta}{CA + A\beta}$ And there is no place in the atmoſphere where the preſſure is the ſame in one condition as the other. But it is, at all heights, greater in the warmer condition (by 5th of this).

If the point Q fall below the central ordinate CK, (fig. 7.) and the ſubtangents βL, βM, be reciprocally as CF, CK, the points β and C will coincide, and B will go off to an infinite height.

If the greater ſubtangent βM be leſs than to be βL as the greater denſity CF to the leſſer CK, β will be found below C, and the ordinate βE is not one of thoſe, by which the denſity of the air, at any height, in either condition, is repreſented. The expreſſion $\frac{CA \times A\beta}{CA - A\beta}$ is more than infinite, the denominator being negative; and there is no height at which the preſſure is the ſame, in both conditions of the atmoſphere; but it is at all heights leſs in the warmer condition (by 5th of ths). But whenever Q falls below the ſuperficial ordinate, whether it be above or below the central ordinate, if βM be *leſs* than to be to βL as AD tc AG, and *greater* than to be to βL as CF to CK, β will fall between C and A, and an equality of preſſure, in both conditions of the atmoſphere, will take place at a finite height, determined as above.

If Q ever falls above the ſuperficial ordinate through A, β will be more above it, and B will be below the earth's ſurface, and there will be no height at which the preſſure will be the ſame;

but it will every where be greater in the warmer temperature.

The calculation for determining $A\beta$ is obvious from the foregoing ſolution of the problem. For putting B for the ſubtangent of the Briggian ſyſtem, D for the difference of the tabular logarithms of AG, AD, and Δ for the difference of thoſe of βL, βM, we have,

$$AR = \frac{\beta L \times \beta M \times D}{B \times LM} \text{ (by prob. 1.)}$$

$$\text{and } R\beta = \frac{\beta L \times \beta M \times \Delta}{B \times LM} \text{ (by cor. 1.)}$$

$$\text{Therefore, } A\beta = \overline{D - \Delta} \times \frac{\beta L \times \beta M}{B \times LM}.$$

Therefore, putting $\beta L = B \pm \frac{p}{q} B$, and $\beta M = B \pm \frac{s}{t} B$. and ſubſtituting $D - \Delta$ for D in the *formulæ* for AL, deduced from problem 1, we ſhall change them into *formulæ* for $A\beta$.

EXAMPLES.

In the example of the preceeding problem $D = 4{,}08$, and $\Delta = 121{,}817$ fathom. Whence $D - \Delta$ is negative, and the point B falls above the ſurface, and the preſſure was no where the ſame, but at all heights greater in the warmer condition.

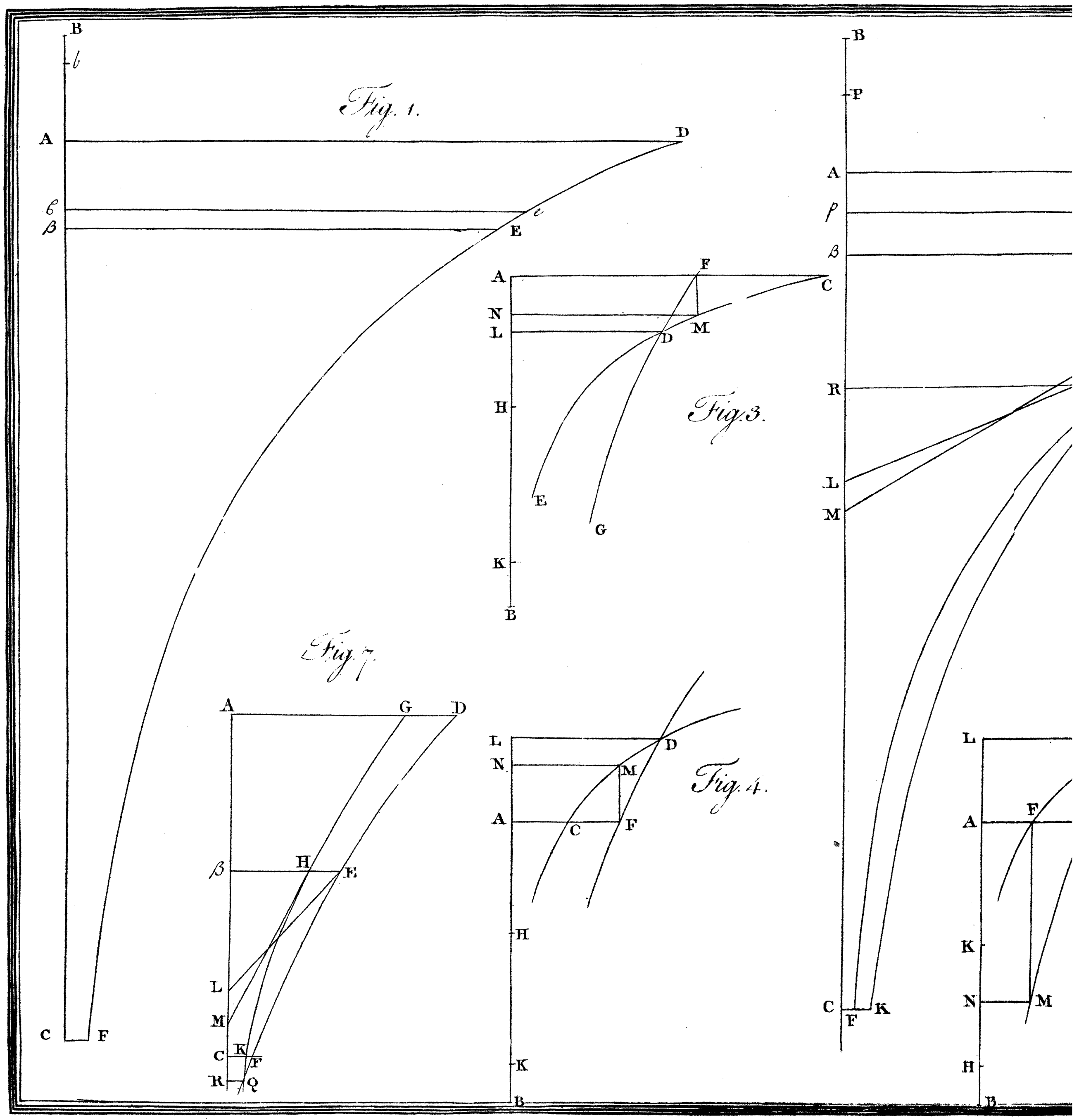


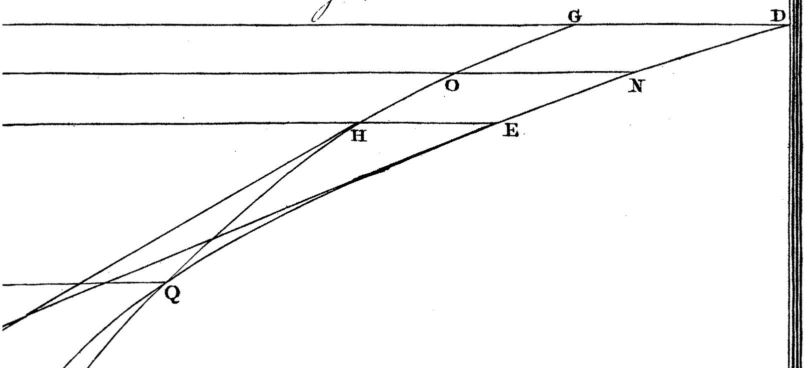
Fig. 1.

Fig. 3.

Fig. 7

Fig. 4.

Fig. 2



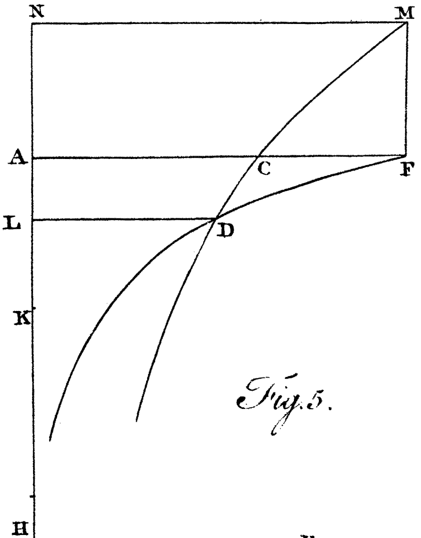


Fig. 5.

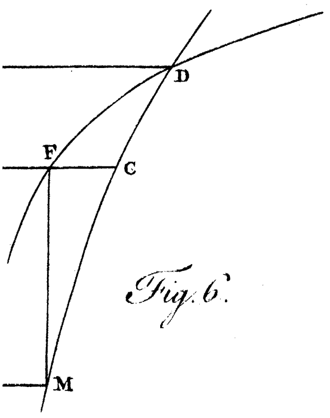


Fig. 6.

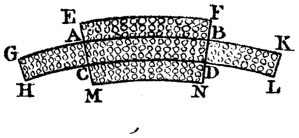


Fig. 8.

Basire sc.

EXAMPLE II.

	d	h	B^{r}. obſerved.	T^{r}. in.	T^{r}. out.	B^{r}. reduced to common temperature	Denſities, by §. 5. prob. 4.
March	26	20	30,07	58	59	30,058	13796,6
	28	21	30,21	54	50	30,21	14138,2

Hence $p=10$. $s=19$. $q=449$. $q+s=468$. $s-p=9$. $\frac{q+s}{s-p}=52$. $D=0.0106238$. $\Delta=0.0084332$. $D-\Delta=0.0021906=21{,}906$ fathom. Therefore, $A\beta=1164{,}42$ fathom $=1{,}32$ miles; and at a height inſenſibly greater, the preſſure was the ſame in both conditions.

In this manner may the heights be determined, in which either the denſity or preſſure of the air, in a given temperature, is the ſame as in any greater or lower temperature; provided the proportion be known between the denſities, in the different temperatures, at the level of the ſea, or at a given elevation above it; otherwiſe both the problems are indeterminate. And I know no method of aſcertaining this proportion, but by actual obſervation of the barometer. The change of temperature determines the proportion in which the ſubtangent of the atmoſpherical logiſtic is changed; but in whatever proportion the ſubtangent of a logiſtic is changed, the ordinate, at a given point of the aſymptote, may be of any imaginable length, or may bear any imaginable proportion to the ſubtangent. There is no given *geometrical* relation, in the nature of the curve, between the length of the one and the other; and experiment hath not yet brought to light any *phyſical* relation in the particular caſe in queſtion. Diligent obſervation of the barometer and thermometer may perhaps, in future, give more ſatisfactory information upon the ſubject. Perhaps I have been too minute in detailing conſequences from an hypotheſis, of what probably never doth obtain; namely, that the atmoſphere may be conſidered as equally heated in all its different parts; and that the variation of the abſolute elaſticity of its component particles is proportional to the change in the degree of uniform temperature, as if the abſolute elaſticity were influenced by no other cauſe. But ſuch is the neceſſary order of enquiry. Theory muſt begin with the moſt ſimple caſes, conſidering what *would* be the effect of ſome known cauſe, acting ſingly, and in the moſt ſimple manner; and comparing the concluſions from what it is *ſuppoſed*, with the effects which *are*, the *difference* leads us to the diſcovery of other cauſes, and to an eſtimation of the ſeparate effects of each, and of the compound effects of all the known cauſes any how combined. I ſhall only add, that as the ſtate of an atmoſphere, unequally heated in its different parts, may be repreſented by parts of different logarithmics, it is poſſible, that inſtead of one point of unaltered denſity, and one point of unaltered preſſure, we might find ſeveral, if ſynchronous obſervations could be made at ſeveral ſucceſſive elevations ſufficiently different.

10. If the atmoſphere of the earth reaches to infinite heights with a finite denſity; for the ſame reaſons, that of Jupiter and every other planet, will reach alſo to infinite heights, above the ſurface of the planet with a finite denſity. The atmoſphere therefore of every planet will reach to the ſurface of every other planet, and to the ſurface of the Sun;

and the atmoſphere of the Sun to the ſurfaces of them all. All theſe atmoſpheres will mingle, and form a common atmoſphere of the whole ſyſtem. This common atmoſphere of the ſyſtem will be infinitely diffuſed, ſince the particular atmoſpheres that compoſe it are ſo. It will reach therefore to every fixed ſtar; and, for the ſame reaſon, that of every fixed ſtar will reach the central body of our ſyſtem, and of every other ſyſtem. The atmoſpheres of all the ſyſtems will mix. The univerſe will have one common atmoſphere, a ſubtle elaſtic fluid, which pervades infinite ſpace, and being condenſed near the ſurface of every larger maſs of matter, by the gravitation towards that maſs, forms its peculiar atmoſphere.

To certain diſtances from every one of theſe great bodies, the condenſations of this infinite fluid will follow the laws of the preceeding theory *nearly*; but to certain diſtances *only*. For that theory conſiders only the effects of the attraction of a ſingle ſphere, and aſſigns the law of the variation of denſity, ſuch as would obtain, if *one* ſpherical body exiſted in the midſt of an infinite fluid; and ſuch as cannot generally obtain, unleſs that hypotheſis were true: for many great ſpheres being immerſed in a common atmoſphere, the attraction of any one, at great diſtances from it, becomes but an inconſiderable part of the whole cauſe, by which the denſity is modified, the joint forces of them all. And in many other circumſtances, beſides the condenſation, the modifications of every particular atmoſphere may depend upon thoſe of others, innumerable and infinitely remote; as the *effluvia* and exhalations of each great maſs, mingling with its atmoſphere, may be diſtributed, in certain proportions, among all the reſt. So that there is probably no branch of phyſics, in which human diſcovery, in its utmoſt extent, muſt always bear ſo ſmall a proportion to what will ſtill remain unknown.

TABLE I.

EQUATION of the boiling point.

Barometer.	Equation.	Difference.
31,0	+ 1,57	
		0,78
30,5	+ 0,79	
		07,9
30,0	0,00	
		0,80
29,5	— 0,80	
		0,82
29,0	— 1,62	
		0,83
28,5	— 2,45	
		0,85
28,0	— 3,31	
		0,86
27,5	— 4,16	
		0,88
27,0	— 5,04	

The numbers in the firſt column to the left expreſs heights of the quickſilver in the Barometer, in Engliſh inches and decimal parts. The ſecond column gives the Equation to be applied, according to the ſign prefixed, to 212° of Bird's Fahrenheit, to find the true boiling point, for every ſuch ſtate of the barometer. The heights of the barometer decreaſe by ½ inches from 31 to 27 inches. The boiling point, for all intermediate ſtates of the barometer, may be found, with a very ſufficient accuracy, by taking proportional parts. For which purpoſe, the differences of tbe equations are given in the third column.

TABLE II.

For the Compariſon of Thermometers.

	LF	PF
1	2,225	2,246
2	4,450	4,493
3	6,674	6,74
4	8,9	8,986
5	11,124	11,233

	L reduced.	P reduced.
80	210,0 —	211,73 —
75	198,86 +	200,50 —
70	187,74 +	189,26 +
65	176,62 —	178,03 —
60	165,49 +	166,80 —
55	154,37 —	155,56 +
50	143,24 +	144,33 —
45	132,12 +	133,09 —
40	121,0 —	121,85 +
35	109,87 +	110,62 +
30	98,75 —	99,39 —
25	87,62 +	88,15 +
20	76,50 +	76,92 +
15	65,38 —	65,69 —
10	54,25 +	54,45 +
5	43,13 —	43,22 +
0	32.	32.
— 5	20,88 —	20,77 —
—10	9,75 +	9,53 +
—15	— 1,37 +	— 1,70 —
—20	— 12,50 —	— 12,93 +
—25	— 23,62	— 24,16 +
—30	— 34,74 +	— 35,40 —
—35	— 45,87 —	— 46,63 +
—40	— 57,00 —	— 57,86 +
—45	— 68,12 —	— 69,10 —
—50	— 79,24	— 80,33
—55	— 90,36 +	— 91,56 +
—60	—101,49 —	—102,80 —
—65	—112,61 +	114,03 —
—70	—123,75 —	125,25 +
—75	—134,87	136,48 +
—80	—146,0 —	147,72 —

	F L	F P
1	0,449	0,445
2	0,899	0,890
3	1,348	1,335
4	1,798	1,780
5	2,247	2,225
6	2,696	2,671
7	3,146	3,116
8	3,595	3,561
9	4,045	4,006
10	4,494	4,451

EXPLANATION.

The principal table conſiſts of three columns. The numbers in the firſt column ſignify degrees, either of DE LUC's ſcale, or the ſcale which hath been hitherto chiefly in uſe among the French, which is divided as M. DE LUC's is; but 28 French inches being generally mentioned, by the French mathematicians, as the mean height of the barometer at PARIS, it is to be ſuppoſed, that their boiling point will generally agree to that ſtate of the barometer. The numbers in the next column to the right, which is marked L *reduced*, give the degrees of BIRD's Fahrenheit correſponding to the degrees of M. DE LUC's ſcale, expreſſed by the numbers of the firſt column; and the numbers in the column marked P *reduced*, give the degrees of BIRD's Fahrenheit correſponding in like manner with the PARIS ſcale. The heights in the firſt column diminiſh by 5° ſucceſſively. But the reduction may be computed to every degree, by means of the little table on the left; which gives the value of ſingle degrees, either of DE LUC's, or of the PARIS ſcale, in degrees of BIRD's Fahrenheit. The titles LF, PF ſignifying DE LUC's degrees, or Paris degrees, reduced to Fahrenheit's, reſpectively.

If there ſhould be occaſion to reduce heights of BIRD's Fahrenheit, either to M. DE LUC's, or the PARIS ſcale, this may be done by the principal table, and the ſmall one on the right hand; which exhibits the value of ſingle degrees of BIRD's Fahrenheit in degrees, both of DE LUC's and the Paris ſcales: the titles FL, FP ſignifying degrees of BIRD's Fahrenheit reduced to M. DE LUC's degrees, or the PARIS degrees, reſpectively.

EXAMPLE.

To find the point upon a BIRD's Fahrenheit, which correſponds to + 58 of DE LUC's ſcale.

By principal table	+ 55 =	154,37
By little table on the left hand	3 =	6,67
Therefore	+ 58 =	161,04

EXAMPLE 2.

To find the point upon the Paris ſcale, which correſponds to + 55 of BIRD's Fahrenheit.

By principal table	+ 54,45 =	10
55—54,45 = 0,55		
By little table on the right hand	0,5 =	0,22
By ditto - - -	0,05 =	0,02
Therefore	+ 55 =	10,24

TABLE III.

EQUATION for the temperature of the QUICKSILVER.

Degrees of Bird's Fahrenh.	Correction of Diff. Logarith.	Fathom	
1	452	0,452	0,003
2	904	0,904	0,006
3	1355	1,355	0,009
4	1807	1,807	0,012
5	2259	2,259	0,016
6	2711	2,711	0,019
7	3163	3,163	0,022
8	3614	3,614	0,025
9	4066	4,066	0,028
10	4518	4,518	0,031
20	9036	9,036	0,062
30	13554	13,554	0,094
40	18072	18,072	0,125
50	22590	22,590	0,156
60	27108	27,108	0,188
70	31626	31,626	0,219

The numbers in the firſt column are to be underſtood to expreſs differences of temperature, in the quickſilver of the portable Barometers at the two ſtations, indicated by Thermometers fixed in the caſes of the Barometers, in degrees of Bird's Fahrenheit. The ſecond column gives the correſponding corrections of the difference of the tabular logarithms of the obſerved heights of the quickſilver in the barometer, to be added or ſubtracted, according as the higher barometer hath been the warmer or cooler of the two. The numbers in the third column are the $\frac{1}{1000}$th parts of thoſe in the ſecond. The manner of uſing them is explained in the general precepts. The fourth column gives the reduction of the obſerved height of the column of quickſilver, in decimal parts of an inch, for the differences of temperature expreſſed by the numbers of the firſt column, when the height of the quickſilver is 30 inches; and the reduction for any other height of the quickſilver may be thus found. Suppoſe the height of the barometer $30 \pm a$ inches, in a certain temperature b, and this is to be reduced to the temperature $b \pm n$. From the fourth column collect the number correſponding to n (by repeated entry, if need be). Call that number M. Then $M \pm \frac{a}{30} M$ is the reduction.

TABLE IV. EQUATION for the Temperature of the AIR.

	40+	50 + 30 —	60 + 20 —	70 + 10 —	80 + 9 —
1	0,0	0,022	0,044	0,067	0,089
2	0,0	0,044	0,089	0,134	0,178
3	0,0	0,067	0,134	0,200	0,267
4	0,0	0,089	0,178	0,267	0,356
5	0,0	0,111	0,223	0,334	0,445
6	0,0	0,134	0,267	0,401	0,535
7	0,0	0,156	0,312	0,468	0,624
8	0,0	0,178	0,356	0,535	0,713
9	0,0	0,200	0,401	0,601	0,802
10	0,0	0,223	0.445	0,668	0,891
20	0,0	0,445	0,891	1,337	1,782
30	0,0	0,668	1,337	2,005	2,673
40	0,0	0,891	1,782	2,673	3,564
50	0,0	1,114	2,228	3,342	4,455
60	0,0	1,337	2,673	4,010	5,347
70	0,0	1,559	3,119	4,678	6,238
80	0,0	1,782	3,564	5,347	7,129
90	0,0	2,005	4,010	6,015	8,020
100	0,0	2,228	4,455	6,683	8,911
200	0,0	4,455	8,911	13,367	17,822
300	0,0	6,683	13,367	20,050	26,733
400	0,0	8,911	17,822	26,733	35,645
500	0,0	11,139	22,278	33,417	44,556
60	0,0	13,367	26,733	40,100	53,467
700	0,0	15,594	31,189	46,783	62,378
800	0,0	17,822	35,645	53,467	71,289
900	0,0	20.050	0.100	60.150	80.200

	1	2	3	4	5	6	7	8	9
1	0,002	0,004	0,007	0,009	0,011	0,013	0,015	0,018	0,020
2		0,009	0,013	0,018	0,022	0,027	0,031	0,036	0,040
3			0,020	0,027	0,033	0,040	0,047	0,053	0,060
4				0,036	0,044	0,053	0,062	0,071	0,080
5					0,055	0,067	0,078	0,089	0,100
6						0,080	0,093	0,107	0,120
7							0,109	0,125	0,140
8								0,142	0,160
9									0,180

In the principal Table, the numbers in the firſt column to the left expreſs fathom. The numbers at the top of each vertical column expreſs degrees of Bird's Fahrenheit above the point 0. The numbers under theſe, in every column, ſhew the corrections correſponding to theſe temperatures, reſpectively, in fathoms and decimal parts, upon ſo many fathom, in an approximate determination of the height in queſtion, as are expreſſed by the numbers in the firſt column to the left. The little Table gives the correction for ſingle degrees of Bird's Fahrenheit. The numbers in the uppermoſt horizontal row, and the outermoſt vertical row, to the left, expreſſing, the one fathom, and the other degrees of the thermometer, indifferently.

GENERAL PRECEPTS, for the CALCULATION of HEIGHTS by theſe TABLES.

FROM the common tables of logarithms, namely, thoſe which exhibit the Briggian logarithms to eight places, write out the logarithms of the numbers, which expreſs the obſerved heights of the quickſilver, in the portable barometers, at the two ſtations.

2. Subtract the leſſer logarithm from the greater.

3. Divide the remainder by 1000. The quotient is a certain number of fathom.

4. Take the difference of the temperatures of the quickſilver, as indicated by the thermometers in the caſes of your portable barometers, and look for the correction correſponding thereto, in the third column of TABLE III. and add that correction to the number laſt found, if the barometer at the higher ſtation hath been the warmer of the two; otherwiſe ſubtract it. Call the ſum in the firſt caſe, the remainder in the latter, the *approximate height*.

5. Add together the temperatures expreſſed at the two ſtations by the thermometers in the open air, under their proper ſigns. Take half the aggregate, and call it the *temperature of the air*.

6. In the principal table of the equation for the temperature of the air (TABLE IV.), look a-top for the decade of degrees next leſs than the temperature of the air, found by the laſt rule; and from the column underneath it (by repeated entries if need be) collect the corrections for the units, decades, and centuries of

fathom in the approximate height; and if you have chiliads of fathom in your approximate height, find the correction for them, by taking the decuple of the correction for the correſponding centuries. Add theſe ſeveral parts of the correction together, and call the ſum N.

7. In the little table look, either a-top or in the outer vertical row to the left, for the units of degrees in the temperature of the air, over and above decades, and collect the correction for theſe, for the units, decades, centuries, &c. of fathom in the approximate height, by taking the numbers found in the little table, and proper multiples thereof. Add theſe numbers together, and call the ſum *n*.

8. If the ſign of the correction found in the great table be +, add $N+n$ to the approximate height; but if the ſign of the correction found in the great table be —, ſubtract $N-n$ from the approximate height. The ſum in the former caſe, the remainder in the latter, is the correct height, according to M. DE LUC's rules.

EXAMPLE.

Barometer.	Tr. in.	Diff.	Tr. out.	Temperature of air.
Lower 29 68	+57	14	+57	+49½
Upper 25,28	+43		+42	

Log. of lower Br.	14724639
of upper	14027771
$\frac{\text{Diff.}}{1000}$	696,868
Equation for ☿	—6,327
Approximate height	690,541
Equation for air N + *n*	+ 14,5
Height correct	705,04

Diff. of Thermometers in = 14° = 10 + 4°.

10° gives	4,519
4° gives	1,808
Equation for ☿	6,327

Temperature of air = 49½ = 40° + 9°½.

N = + 0

9° × 600 fathom gives	12	by little table for air.
9° × 90	1,8	
½ × 600	0,6	
½ × 90	0,1	

n = 14,5

N + *n* = 14,5

EXAMPLE II.

Barometer.	Tr. in.	Diff.	Tr. out.	Temperature of air.
Lower 29,45	+38	3	+31	33
Upper 26,82	+41		+35	

Log. of lower	1.4690853
of upper	1.4295908
$\frac{\text{Diff.}}{1000}$	394,945
Equation for ☿	+ 1,356
Approximate height	396,301
Equation for Air — $\overline{\text{N}-n}$.	— 6,182
Height correct	390,119

Diff. of Trs. in = 3° gives equation for ☿ 1,356.

Temperature of air = 33° = 30° + 3°.

30° upon 300 fathom gives	6,683	by great table for air. sign —
upon 90	2,005	
upon 6	0,134	

N = 8,822

3° × 300 fathom gives	2,000	by little table for air.
× 90	0,600	
× 6	0,04	

n = 2,640

N — *n* = 6,182

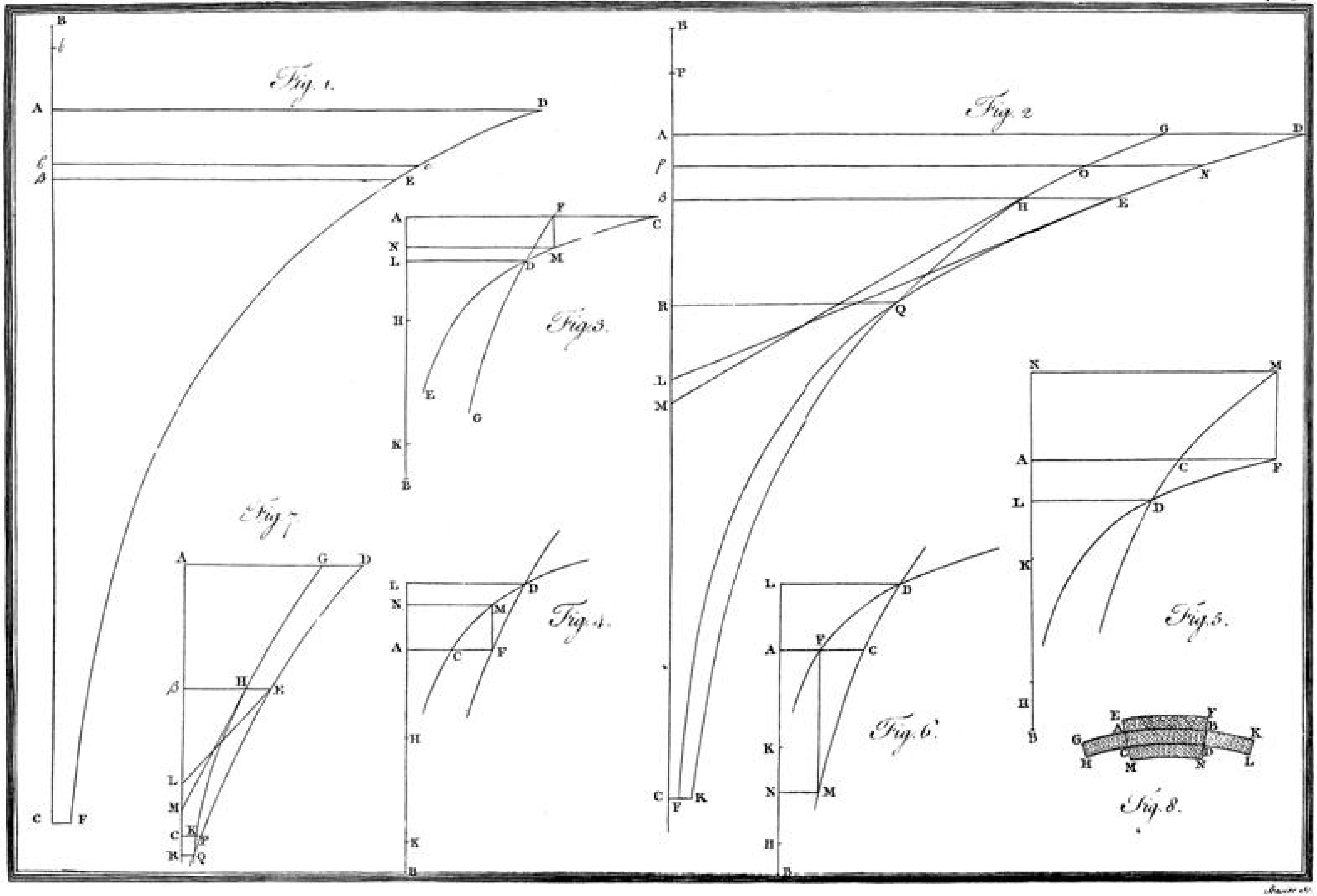


Fig. 1.
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Fig. 2
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Fig. 3.
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Fig. 4.
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Fig. 5.
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Fig. 6.
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Fig. 7
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Fig. 8.
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Basire sc.